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# ANALYSIS OF EQUIVALENT APERTURE OF CURVILINEAR ANTENNA ARRAYS

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# АНАЛІЗ ЕКВІВАЛЕНТНОЇ АПЕРТУРИ КРИВОЛІНІЙНИХ АНТЕННИХ РЕШІТОК

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# АНАЛИЗ ЭКВИВАЛЕНТНОЙ АПЕРТУРЫ КРИВОЛИНЕЙНЫХ АНТЕННЫХ РЕШЕТОК

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**Abstract.** The article presents the results of the analysis of equivalent aperture of curvilinear antenna arrays on the basis of the obtained mathematical models. The equivalent aperture of the curvilinear antenna array is a projection of the arc on the plane, which is perpendicular to the maximum emission direction or to the direction of electromagnetic waves receiving. Feature of the curvilinear antenna array is the antenna elements arrangement along the arc of the ellipse with different orientations and arbitrary parameters. The dependencies of the equivalent aperture curvilinear antenna array of scanning angle were obtained. It is reported the possibility of increase the equivalent aperture of curvilinear antenna array in given sector of scanning angles.

**Key words:** curvilinear antenna array, equivalent aperture, arc of ellipse, sector of scanning angles.

**Анотація.** У статті наведені результати аналізу еквівалентної апертури криволінійних антенних решіток на основі отриманих математичних моделей. При цьому під еквівалентною апертурою криволінійної антенної решітки розуміється проекція дуги на площину, що перпендикулярна напряму максимального випромінювання або напряму приходу електромагнітних хвиль. Особливістю криволінійної антенної решітки є розташування антенних елементів уздовж дуги еліпса з різною орієнтацією та довільними параметрами. Отримано залежності еквівалентної апертури криволінійної антенної решітки від кута сканування. Показана можливість збільшення еквівалентної апертури криволінійної антенної решітки в заданому секторі кутів сканування.

**Ключові слова:** криволінійна антенна решітка, еквівалентна апертура, дуга еліпса, сектор кутів сканування.

**Аннотация.** В статье представлены результаты анализа эквивалентной апертуры криволинейных антенных решеток на основе полученных математических моделей. При этом под эквивалентной апертурой криволинейной антенной решетки понимается проекция дуги на плоскость, перпендикулярную направлению максимального излучения или направлению прихода электромагнитных волн. Особенностью криволинейной антенной решетки является расположение антенных элементов вдоль дуги эллипса с различной ориентацией и произвольными параметрами. Получены зависимости эквивалентной апертуры криволинейной антенной решетки от угла сканирования. Показана возможность увеличения эквивалентной апертуры криволинейной антенной решетки в заданном секторе углов сканирования.

**Ключевые слова:** криволинейная антенная решетка, эквивалентная апертура, дуга эллипса, сектор углов сканирования.

Problems of the theory and technology of antenna devices are of particular interest in terms of improving the efficiency of the radiotechnical and wireless telecommunication systems. This is due to the fact that the antenna device is part of the radiochannel and the antenna device determines largely the potential of a radiosystem in general. Among the various types of antenna devices, undoubtedly the antenna arrays form the separate perspective group. These types of antennas are used for solving specific tasks in different modes of the emission and reception. Using antenna arrays allows realizing the specific functions in radiotechnical and wireless telecommunications systems. However, more and more obvious is problem of development of multifunctional systems, in which combined several functions. This imposes certain requirements on the antenna arrays, which must satisfy the advanced set of requirements and be multifunctional devices.

The modern theory of antennas has the significant amount of developments on the antenna arrays and characterized by significant using of digital signal processing techniques [1,2]. New approaches resulted in the emergence of the difficulties that do not give answers from the developed theory of antennas, and therefore the prospective and current scientific and technical problems of the antenna arrays require rapid solutions.

An example of using of an antenna arrays in modern wireless telecommunication systems, in particular, the standards of the fourth generation mobile communication LTE (Long Term Evolution) [3] and LTE Advanced [4], is the technology of MIMO (Multiple Input Multiple Output) and its extended version of Multi-user MIMO or MU-MIMO. This technology involves the use of multiple antennas or antenna arrays as at the receiving side and at the transmission side, and a diverse set of processing algorithms radiosignals [3,5,6].

The main advantages of MIMO technology are the following factors.

- Reducing fading at a receiver input due to the implementation of different kinds of a diversity;
- Increase of capacity due to spatial separation and combining the channels (space multiplexing);
- Increase the signal-noise ratio at a receiver input due to the signals coherent summation at transmit and/or receive sides of a radio link (beamforming).

All of these advantages of MIMO technology are realized through signal processing algorithms and due to the potential properties of used antenna devices. However, for the antenna devices and their properties by the MIMO technology implementation are given little attention. Probably therefore [7] the optimistic estimates of performance MIMO and mobile system standards LTE и LTE Advanced are obtained.

Potential properties of an antenna arrays defined the spatial arrangement of the antenna elements and their number, the type of the antenna element, and their orientation, the amplitude and phase excitation of the antenna elements. Linear antenna arrays and antenna arrays when antenna elements positioned along circle or along the arc of the circle investigated in detail [8, 9]. However, new potential can be expected from curvilinear antenna array [10, 11]. The purpose of this work was to analyze the potential properties of the antenna arrays when antenna elements are arranged on a curved surface on positions of the angular change their equivalent aperture.

The case of an antenna elements arrangement on an arch of the second order, in particular on an ellipse arch, is of a special interest of antenna array configuration of the arc antenna array from an equivalent aperture angle changing position. The ellipse is described by the following parameters: $a, b$ are the big and small half-axis of an ellipse; $ex$ is an excentricity of the ellipse ($0 \le ex < 1$). The interrelation between these parameters is defined on the basis of expressions:

$$ex = \sqrt{1 - b^2/a^2}\ ;\ b = a\sqrt{1 - ex^2}\ . \qquad (1)$$

Schematic images of an arc antenna array with antenna elements arrangement on an ellipse arch are shown on fig. 1.

In particular, the arc antenna array with antenna elements arrangement on an arch, which is based on a bigger axis of the ellipse $D_a = 2a$, with the excentricity of the ellipse $ex = 0{,}866$ ($a = 2b$) and with the excentricity of the ellipse $ex = 0$ ($a = b$ is a semi-circle arch) is schematically represented on the fig.1,a. The arc antenna array with antenna elements arrangement on the arch, which is based onsmall axis of the ellipse $D_b = 2b$, with the excentricity of the ellipse $ex = 0{,}866$ ($a = 2b$) and with the excentricity of the ellipse $ex = 0$ ($a = b$ is a semi-circle arch) is schematically represented on fig. 1,b.

Angular coordinates of the antenna elements, which are equal located along the corresponding axis of the ellipse $D_b = 2b$ or $D_a = 2a$ and with given parameters $a, b$ and $ex$, are determined according to expressions:

$$\theta_{e_i} = \frac{\pi}{2}\left[1 - \frac{2(i-1)}{N-1}\right]; \quad \varphi_{e_i} = \frac{\pi}{2} - \theta_{e_i}, \tag{2}$$

where $i = 1...N$; $N$ is total number of the antenna elements.

The directions of counting out angle coordinates $\theta_{e_i}$, $\varphi_{e_i}$ are shown on fig.1.

On the basis of (2), and also the geometrical description of the ellipse (1), the relative coordinates of the antenna elements $x_{e_i}/\lambda$, $y_{e_i}/\lambda$ in rectangular coordinates system are defined on the basis of the following expressions.

At antenna element arrangement on bigger axis of the ellipse $D_a = 2a$ (see fig. 1,a), these expressions are

$$x_{e_i}/\lambda = R_a\left(\varphi_{e_i}\right)/\lambda \cos\varphi_{e_i}; \quad y_{e_i}/\lambda = R_a\left(\varphi_{e_i}\right)/\lambda \sin\varphi_{e_i}, \tag{3}$$

where
$$R_a\left(\varphi_{e_i}\right) = \frac{a\sqrt{1-ex^2}}{\sqrt{1-ex^2\cos^2\varphi_{e_i}}} = \frac{D_a\sqrt{1-ex^2}}{2\sqrt{1-ex^2\cos^2\varphi_{e_i}}}. \tag{4}$$

At antenna element arrangement on smaller axis of the ellipse $D_b = 2b$ (see fig. 1,b), these expressions are

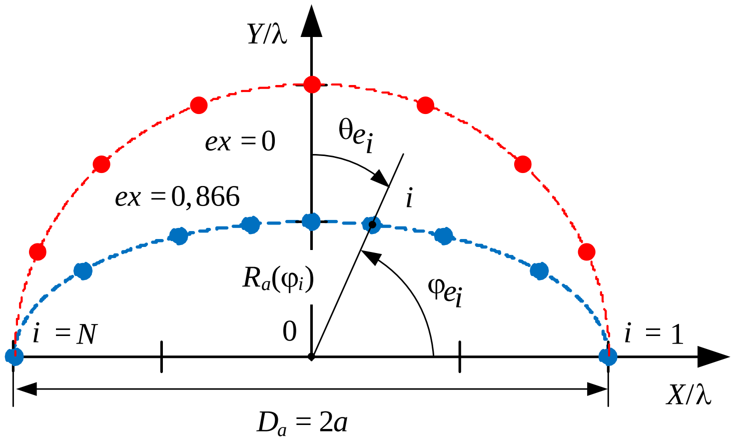
$$x_{e_i}/\lambda = R_b\left(\varphi_{e_i}\right)/\lambda \cos\varphi_{e_i}; \quad y_{e_i}/\lambda = R_b\left(\varphi_{e_i}\right)/\lambda \sin\varphi_{e_i}, \tag{5}$$

where
$$R_b\left(\varphi_{e_i}\right) = \frac{b}{\sqrt{1-ex^2\sin^2\varphi_{e_i}}} = \frac{D_b}{2\sqrt{1-ex^2\sin^2\varphi_{e_i}}}. \tag{6}$$

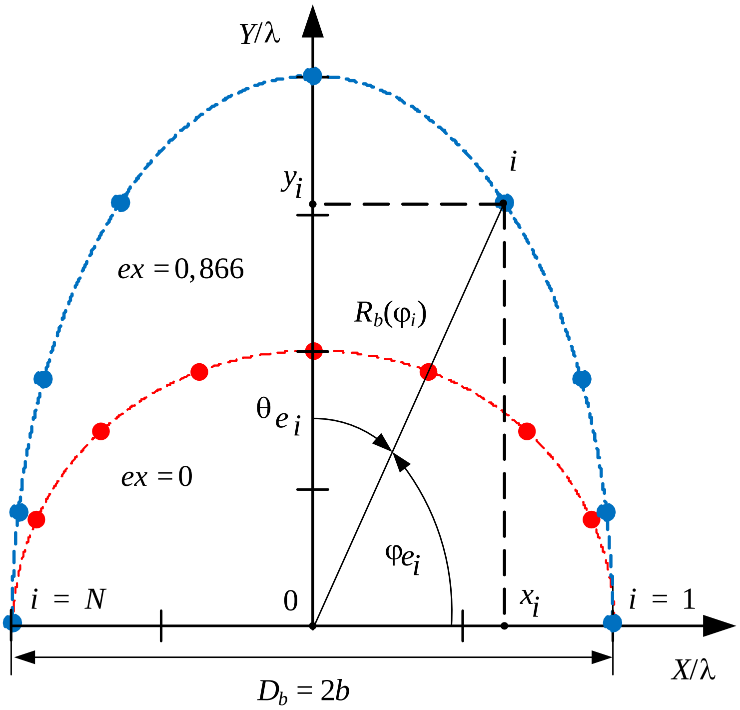
It is important to estimate the equivalent aperture depending of the scanning angle $\theta$, because the main mode of radiation (receive) of the researched antenna array is the scanning mode in given angle sector.

The equivalent aperture of the curvilinear antenna array is a projection of the arc on the plane, which is perpendicular to the maximum emission direction $\theta_{\max}$ or to the direction of electromagnetic waves receiving.

Curvilinear antenna array equivalent aperture $D_{eq}$ with antenna element arrangement on the arch of ellipse generally depends on geometrical parameters of the arch of the ellipse $D_b = 2b$ or $D_a = 2a$, and also the excentricity of the ellipse $ex$ and the scanning angle $\theta$.




a)




b)

Figure 1 – Different types of curvilinear antenna arrays:
a) on the ellipse arc $D_a = 2a$, $ex = 0{,}866$, and $ex = 0$;
б) on the ellipse arc $D_b = 2b$, $ex = 0{,}866$, and $ex = 0$

On the base of fig. 2,a, the equivalent aperture of the curvilinear antenna arrays with antenna element arrangement on bigger axis of the ellipse $D_a = 2a$ can be calculated by formula:

$$D_{eq} = \left[ R_a(\varphi) + R_a^*(\varphi) \right] \cos\gamma \,, \qquad (7)$$

where $R_a(\varphi)$ is calculated according to (4).

According to fig. 2,b and sine theorem

$$R_a^*(\varphi) = -a\cos\theta\cos^{-1}(\theta+\varphi) = 0{,}5\,D_a\cos\theta\cos^{-1}(\theta+\varphi).$$

$\gamma = \varphi - \zeta$, $\zeta = \pi - \theta$ according to fig. 2,c.

After that $\cos\gamma = -\cos(\theta+\varphi)$ and expression (7) converts to

$$D_{eq}(\theta) = 0{,}5D_a\cos\theta - R_a(\varphi)\cos(\theta+\varphi). \quad (8)$$

The equation of normal to the point, which is located on an ellipse can used for determination of dependence between angles $\theta$ and $\varphi$. According to this equation and using (1) and (3), can get

$$\tan\zeta = \left(1-ex^2\right)^{-1}\tan\varphi. \quad (9)$$

After conversion (9) taking into account $\zeta = \pi - \theta$, and also a range of function definition of the tangent can get:

$$\begin{aligned} &\varphi = \operatorname{atan}\left[\left(ex^2-1\right)\operatorname{tg}\theta\right] \quad \text{at } \theta = -\pi/2\ldots 0;\\ &\varphi = \pi + \operatorname{atan}\left[\left(ex^2-1\right)\operatorname{tg}\theta\right] \quad \text{at } \theta = 0\ldots\pi/2. \end{aligned} \quad (10)$$

Similar it is possible to receive the dependence of arc antenna array equivalent aperture at antenna element arrangement on smaller axis of the ellipse $D_b = 2b$ (see fig. 3).

Omitting intermediate transformations can get

$$D_{eq}(\theta) = 0{,}5D_b\cos\theta - R_b(\varphi)\cos(\theta+\varphi), \quad (11)$$

where $R_b(\varphi)$ is calculated according to (6);

$$\begin{aligned} &\varphi = \operatorname{atan}\left[\left(ex^2-1\right)^{-1}\operatorname{tg}\theta\right] \quad \text{at } \theta = -\pi/2\ldots 0;\\ &\varphi = \pi + \operatorname{atan}\left[\left(ex^2-1\right)^{-1}\operatorname{tg}\theta\right] \quad \text{at } \theta = 0\ldots\pi/2. \end{aligned} \quad (12)$$

In that specific case, when the arch of the ellipse is a semi-circle at $ex = 0$, and the corresponding axis of the ellipse $D_b = 2b$ or $D_a = 2a$ is diameter $D$ of a circle, expressions (10) and (12) are transformed to $\varphi = \pi - \theta$, and expressions (8) and (11) are transformed to

$$D_{eq}(\theta) = 0{,}5D\left(1+\cos\theta\right). \quad (13)$$

The dependences $D_{eq}(\theta)$, which are calculated by (8), (10) and (11), (12) for cases of the arc antenna arrays with antenna element arrangement of along the arch, based on bigger and smaller axis of the ellipse at $ex = 0{,}866$, $ex = 0{,}836$ and $ex = 0$ (see fig. 1,a, and 1,b) at identical overall dimensions are represented on fig. 4.

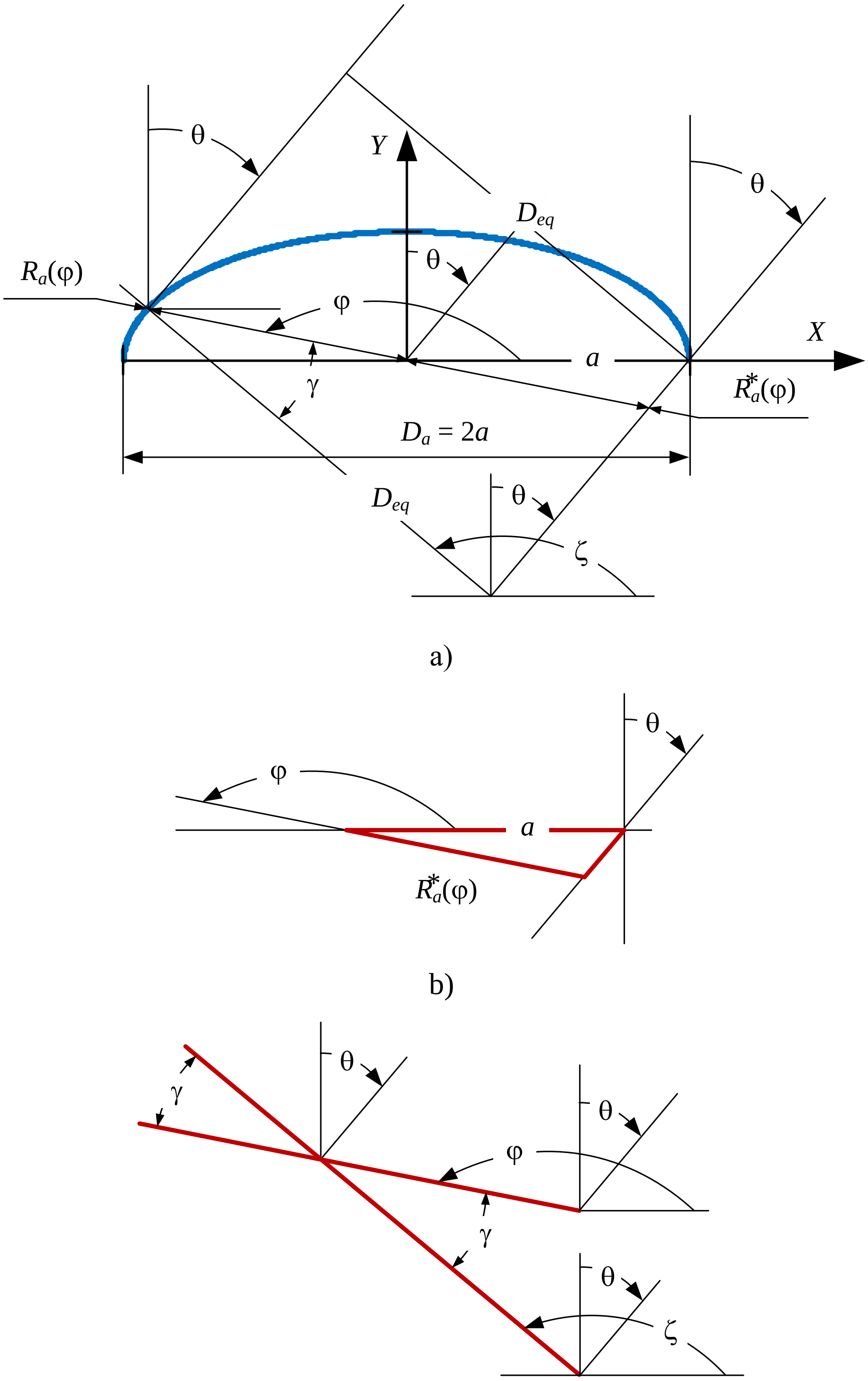



Figure 2 – To definition of arc antenna array equivalent aperture at antenna element arrangement on bigger axis of the ellipse: a) general view; b) for calculation of $R_a^*(\varphi)$; c) for calculation of $\gamma$

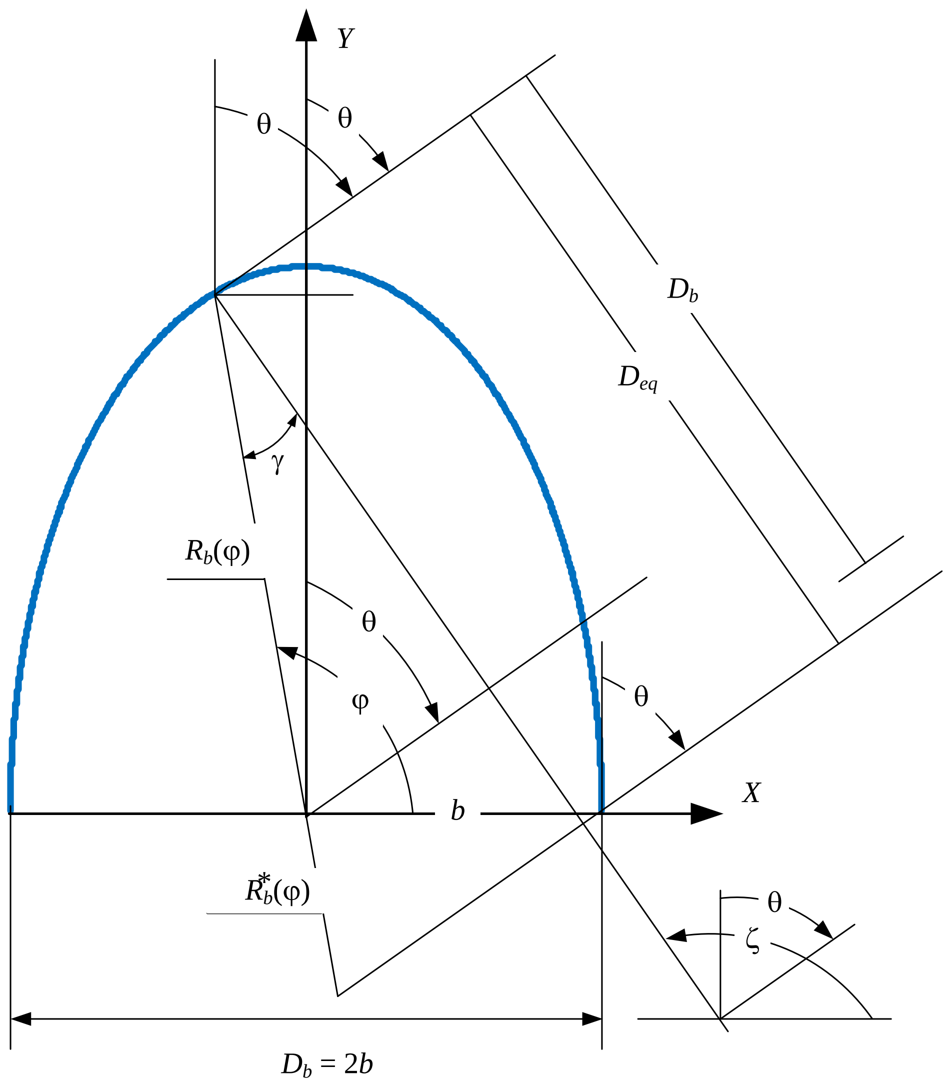



Figure 3 – To definition of arc antenna array equivalent aperture
at antenna element arrangement on smaller axs of the ellipse

Increasing the arc antenna array equivalent aperture in comparison with equivalent aperture of the linear antenna array can see according to the got dependences. Proceeding from a criterion of the maximum equivalent aperture in sector of scanning angles from 0 to the 90 deg. it is possible to allocate arc antenna array with antenna element arrangement on smaller axis of the ellipse $D_b = 2b$ (see fig. 4, a curve 4). In this case, the excentricity of the ellipse is equal $ex = 0{,}866$, that corresponds to the equality of cross $D_b = 2b$ and longitudinal $a$ overall arc antenna array dimensions. The maximum excess of arc antenna array equivalent aperture $D_{eq}$ over cross overall dimension $D_b = 2b$ on 15,5% in the direction of the $\theta = 55$ deg. is thus observed (see fig. 3). Quantitative estimates $D_{eq}$ show an increasing in comparison with $D_b = 2b$: for more than 5% in the sector of scanning angles $\theta = 20...84$ deg.; for more than 10% in a sector of scanning angles $\theta = 31...76$ deg.; for more than 15% in a sector of scanning angles $\theta = 48...61$ deg.

For the arc antenna array with antenna element arrangement along the arch of the ellipse, based on smaller axis of the ellipse $D_b = 2b$, at reduction of an excentricity of the ellipse to $ex = 0{,}836$ (see fig. 4, a curve 5), that is equivalent to reduction of longitudinal overall dimension $a$

of this arc antenna array for 9% and this leads to a change of arc antenna array equivalent aperture in sector of scanning angles from 0 to the 90 deg. no more than for 9% relatively $D_b = 2b$.

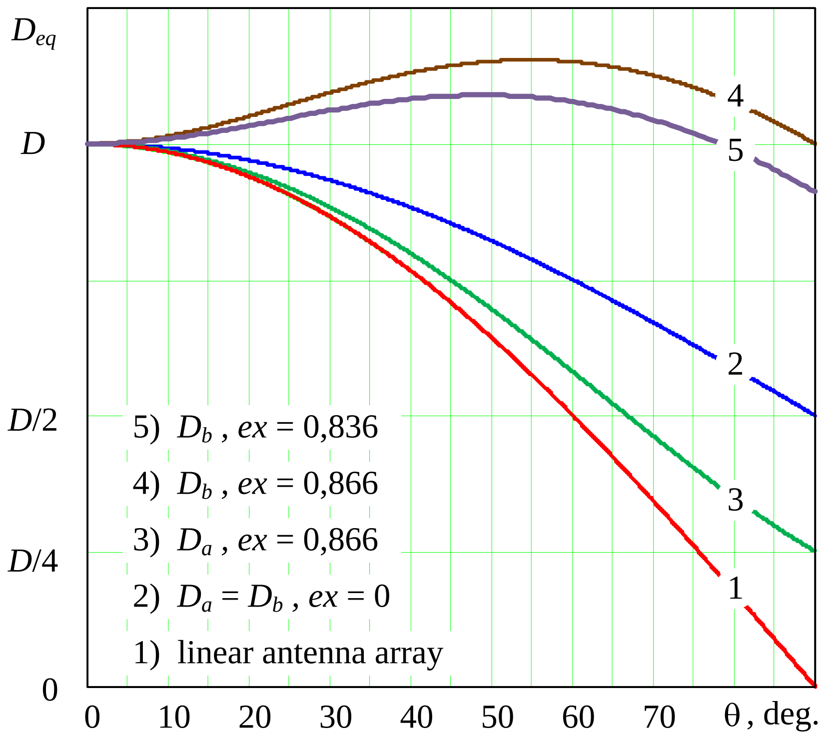



Figure 4 – Dependences of arc antenna array equivalent aperture from scanning angle $\theta$:

1) linear antenna array;
2) on the arc of circle $D_a = D_b$, $ex = 0$;
3) on the arc of ellipse $D_a = 2a$, $ex = 0{,}866$;
4) on the arc of ellipse $D_b = 2b$, $ex = 0{,}866$;
5) on the arc of ellipse $D_b = 2b$, $ex = 0{,}836$

Thus, on the basis of the conducted researches it is possible to state the following.

1) The configurations of the curvilinear antenna array with antenna elements arrangement along an arch of the ellipse, based on bigger and smaller axes of the ellipse were offered.

2) The new mathematical models of curvilinear antenna array equivalent aperture at antenna elements arrangement of both along an ellipse arch were got. These models allow to analyze angular changes of antenna array equivalent aperture in the given sector of scanning angles $\theta$, and also they allow to unite on the basis of the general approaches linear and semi-ring antenna arrays, ellipse antenna arrays with arbitrary values of the ellipse excentricity $0 \le ex < 1$.

3) Qualitative and quantitative analysis of angular change of antenna array equivalent aperture was carried out. There is shown an efficiency of the curvilinear antenna array with antenna elements arrangement along an ellipse arch, based on small axes of the ellipse, from positions of bigger equivalent aperture in a sector of scanning angles from 0 to the 90 deg.

The results of this research can be used to create optimal antenna arrays. Further studies may be directed to the search for other configurations of antenna arrays and to justification of required number of the antenna elements.

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