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DIAGONALIZATION OF THE DIFFERENTIAL OPERATOR MATRIX IN THE CASE OF THE MULTIDIMENSIONAL CIRCUITS

ДІАГОНАЛІЗАЦІЯ МАТРИЦІ ОПЕРАТОРІВ ДИФЕРЕНЦІЮВАННЯ БАГАТОМІРНИХ КІЛ

ДИАГОНАЛИЗАЦИЯ МАТРИЦЫ ОПЕРАТОРОВ ДИФФЕРЕНЦИРОВАНИЯ МНОГОМЕРНЫХ ЦЕПЕЙ

Summary. In the present paper we propose the explicit mathematical method that is assumed to be quite simple from the engineering and applied aspects and that allows to diagonalize an n-dimensional system of the arbitrary partial differential operator equations over the space $\mathbf{R}^m$. All sought for scalar equations have an only one component of the unknown n-dimensional vector function $\vec{F}(x_1, \dots, x_m)$ and are obtained by the application of the corresponding partial differential operators to the original system equations. These operators are the initial matrix elements and the only one requirement is their commutativity in pairs.

Анотація. В роботі пропонується гостро математичний та досить простий з інженерно-прикладної точки зору конструктивний метод діагоналізації системи n-го порядку довільних диференціальних операторних рівнянь у часткових похідних над простором $\mathbf{R}^m$. Шукані скалярні рівняння , кожне з яких містить тільки одну компоненту невідомої n-мірної вектор-функції $\vec{F}(x_1, \dots, x_m)$, одержані послідовним застосуванням відповідних диференціальних операторів у часткових похідних – елементів вихідній матриці, до рівнянь первісної системи. При цьому єдиною вимогою для даних операторних матричних елементів являється їх комутативність парами.

Аннотация. В работе предлагается строго математический и достаточно простой с инженерно-прикладной точки зрения конструктивный метод диагонализации системы n-го порядка произвольных дифференциальных операторных уравнений в частных производных над пространством $\mathbf{R}^m$. Искомые скалярные уравнения, каждое из которых содержит ровно одну компоненту неизвестной n-мерной вектор-функции $\vec{F}(x_1, \dots, x_m)$, получены последовательным применением соответствующих дифференциальных операторов в частных производных – элементов исходной матрицы, к уравнениям первоначальной системы. При этом единственным требованием для упомянутых операторных матричных элементов является их попарная коммутативность.

It is well known that the majority of real physical processes may be described either by the partial differential equations (PDEs) with the constant coefficients or by their systems. That is the main reason why even now the problem of the construction of the explicit and simple mathematical procedure for the solution of the various PDEs' systems remains quite urgent in applied mathematics, physics and engineering [1 – 3]. The classical approach in this direction is the integral transformation method jointly with the generalized function theory [2, 4, 5]. In the mentioned case the investigator must choose the correct integral transformation not only from the mathematical point of view but also taking into account the physical statement of the original problem.

Systems of PDEs with the constant coefficients are broadly used in the multidimensional circuit theory (look, e. g. [6]). The interest to this topic has grown to a considerable extent due to the appearance of the multidimensional wave digital filters [7]. The multidimensional analogous circuits are used as the prototype during the above mentioned filters' synthesis. Rather important results [8] were obtained in classical electrodynamics [9] because of some statements of the multidimensional analogous circuit theory. In this case Maxwell axioms were considered as the system of PDEs. In [10 – 12] the diagonalization problem was solved for the system of the differential Maxwell equations that was represented in various initial forms. Each diagonalization procedure of the original matrix was done by the consistent application of the appropriate operators to the equations of the initial system. The result of this procedure was the system of scalar equations and it was equivalent to the original one. The mentioned approach is the considerable simplification of the solution for the arbitrary system of PDEs, since this object is reduced to the equivalent one whose equations are scalar and depend on the only one component of the unknown vector function.

Nevertheless, as far as it is known, the diagonalization problem of the PDEs' system for the multidimensional circuits is not found yet in the general case. Therefore, the main purpose of the given paper is the construction of such diagonalization method that deals without the inverse matrix differential operator.

1. The problem statement. Turning towards the generalized functions' method we may notice that this approach, though is very elegant mathematically, remains rather difficult for the nonmathematicians at the stage of its explicit realization. Thus, for example, in the monograph [2, p. 127 – 261], even in the simple case of the differential operator polynomials with constant coefficients

$$Pu(x_1,\ ...,x_n)=\sum_{k=1}^{n}a_k\partial_k u,\quad \partial_k=\frac{\partial}{\partial x_k},\quad \forall a_k=\text{const}\in \mathrm{R}\quad (k=\overline{1,n}) \tag{1}$$

the solution of the following equation

$$Pu(x_1,\ ...,x_n)=f(x_1,\ ...,x_n) \tag{2}$$

is reduced to the expansion of operator P by its eigenfunctions; u and f in formula (2) are the appropriate n-dimensional unknown and given vector-functions that have continuous derivatives in some domain of the space $\mathbf{R}^n$. Such approach brings the operator P from (1) to the diagonalized form, i.e. the original equation (2) is also diagonalized.

The principal obstacle of the applied character here is that again from the very beginning the investigator researcher has to deal with integral transformations which operate in various classes of the generalized functions, i.e.: basic, moderate and quickly increased. Hence, when an engineer even in the system diagonalization procedure uses the both of the recently mentioned methods, he must be very acute in all their mathematical details. Moreover, for every specific class of the systems these approaches are realized in their own way.

With the problem of the PDEs first order systems' solution over the space (x, y, z, t) the authors have come across when they studied the classical electrodynamical objects and Maxwell equations, — in particular. Thus, in [8] was shown that in the case of classical electrodynamics axiomatic construction for the linear homogeneous isotropic undisturbed media with the outside currents two main postulates as the following vector equations may be sufficiently accepted

$$\begin{aligned} rot\vec{H}&=y\vec{E}+e_a\partial_0\vec{E}+\vec{j}^{\,\text{ст}},\\ \text{-}rot\vec{E}&=м_a\partial_0\vec{H}, \end{aligned} \tag{3}$$

where: $\vec{E}=\vec{E}(x,y,z,t)$ and $\vec{H}=\vec{H}(x,y,z,t)$ are the unknown vector-functions of the electric and magnetic field tension; differential operator $\partial_0=\dfrac{\partial}{\partial t}$; the given vector-function $\vec{j}^{\,\text{ст}}=\vec{j}^{\,\text{ст}}(x,y,z,t)$ describes the outside current sources; the positive constants $\sigma, \mu_a, \varepsilon_a$ are the specific conductivity, absolute permeance and dielectric permeability correspondingly.

Further, in [11] the vector system (3) was reduced to the equivalent system of six PDEs, and every equation had an only one unknown scalar vector component of $\vec{E}=\{E_i\}_{i=1}^{3}$ or $\vec{H}=\{H_i\}_{i=1}^{3}$. In other words, the original matrix system was diagonalized. This result was obtained by the consistent application of the corresponding differential operators to six original equations of system (3) that were written in terms of $\vec{E}=\{E_i\}_{i=1}^{3}$ and $\vec{H}=\{H_i\}_{i=1}^{3}$ respectively.

In given paper we propose the generalization and analytical formalization of the previous results [10], [11] in the case of arbitrary n-dimensional differential operator equation systems over the space $\mathbf{R}^m$

$$\sum_{i=1}^{n}A_{ji}F_i=f_j\quad (j=\overline{1,n}), \tag{4}$$

where: $\vec{F}=\vec{F}(x_1,...,x_m)$ and $\vec{f}=\vec{f}(x_1,...,x_m)$ are the n-dimensional unknown and given vector-functions that are $n\cdot s$ continuously differentiated in some domain of the space $\mathbf{R}^m$; s is equal to the maximum order of the higher operator A_{ji} derivative for all $j,i=\overline{1,n}$, and partial differential operators A_{ji} are utterly arbitrary. The only requirement of the proposed diagonalization procedure is their commutativity in pairs

$$A_{ji}A_{kl}=A_{kl}A_{ji}\quad (j,i,k,l=\overline{1,n}), \tag{5}$$

where the consistent operator application is defined as usual from the right to the left, i.e. from the "inner" to the "external".

We should remind now that the diagonalization of system **(**4**)** is treated here in the same meaning as earlier. The initial system is reduced to the equivalent one that consists of n scalar partial differential operator equations and each equation has an only one component $F_i = F_i\ (x_1,...,x_m)\ \ (i=\overline{1,n})$ of the unknown vector-function $\vec{F}$. This result is also obtained by the consistent application of the appropriate partial differential operators to the original equations of system **(**4**)**.

The proposed procedure is generalized for matrices with the operator-block structure and is demonstrated in the case of Maxwell vector system (3).

2. Reduction of the first initial system equation to a scalar one ("upward" diagonalization step). At the first diagonalization stage we raise a problem to obtain the scalar equation regarding one of the unknown components $\{F_i\}_{i=1}^{n}$. Not breaking the common character of our results, we assume that the sought for component is F_1.

Step 1. We separate the last equation of system **(**4**)** and isolate the item with scalar F_n in all n equations of the considered system:

$$\begin{cases} \sum\limits_{i=1}^{n-1} A_{ji}F_i + A_{jn}F_n = f_j \quad (j=\overline{1,n-1}), \\ \sum\limits_{i=1}^{n-1} A_{ni}F_i + A_{nn}F_n = f_n. \end{cases} \tag{6}$$

Then we apply to the last equation of (6) the operator

$$\left(-A_{jn}\right) \quad (j=\overline{1,n-1}) \tag{6$'$}$$

consistently for all j from (6′), and to the remained $n-1$ equations of the same system the following operator

$$A_{nn}, \tag{6$''$}$$

is applied. Afterwards we sum consistently the last transformed nth equation and the rest $n-1$ transformed equations for all $j=\overline{1,n-1}$. As the result we come to the system that is equivalent to (6), its equations from the first till the $(n-1)$th have no anymore the scalar F_n, and the nth equation is separate

$$\begin{cases} \sum\limits_{i=1}^{n-1} \left(A_{nn}A_{ji} - A_{jn}A_{ni}\right)F_i = A_{nn}f_j \ - A_{jn}f_n \quad (j=\overline{1,n-1}), \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \sum\limits_{i=1}^{n-1} A_{ni}F_i + A_{nn}F_n = f_n. \end{cases} \tag{7}$$

Such separate equations that close the appropriate system at every diagonalization step further in this paper we shall call "the single equations".

Introducing the auxiliary notations for the given operators and functions

$$\begin{aligned} &A_{nn}A_{ji} - A_{jn}A_{ni} = B_{ji}^{(1)} \quad (j,i=\overline{1,n-1}), \\ &A_{nn}f_j - A_{jn}f_n = g_{j1} \quad (j=\overline{1,n-1}), \end{aligned} \tag{8}$$

we consider now only the first $n-1$ equations from (7), i.e. the subsystem of (7) that looks like

$$\sum_{i=1}^{n-1} B_{ji}^{(1)}F_i = g_{j1} \quad (j=\overline{1,n-1}), \tag{9}$$

and that is the system (7) without its "single equation".

Step 2. In all $n-1$ equations of (9) we isolate the item with the component F_{n-1}, and the last system equation is written separately

$$\begin{cases} \sum\limits_{i=1}^{n-2} B_{ji}^{(1)}F_i + B_{j,n-1}^{(1)}F_{n-1} = g_{j1} \quad (j=\overline{1,n-2}), \\ \sum\limits_{i=1}^{n-2} B_{n-1,i}^{(1)}F_i + B_{n-1,n-1}^{(1)}F_{n-1} = g_{n-1,1}. \end{cases} \tag{10}$$

Then we apply to the last equation of (10) the operator

$$\left(-B^{(1)}_{j,n-1}\right) \quad (j=\overline{1,\,n-2}) \tag{10'}$$

consistently for all *j* from (10′), and to the remainded $n-2$ equations of (10) the operator

$$B^{(1)}_{n-1,n-1} \tag{10''}$$

is applied. Afterwards we sum the transformed ($n-1$)th equation and the rest transformed $n-2$ equations in the consecutive order for all $j=\overline{1,\,n-2}$. As the result we come to the system that is equivalent to (10), its equations from the first till the ($n-2$)th do not contain now two scalar functions F_{n-1}, F_n, and the ($n-1$)th equation is "single":

$$\begin{cases} \sum\limits_{i=1}^{n-2}\left(B^{(1)}_{n-1,n-1}B^{(1)}_{ji} - B^{(1)}_{j,n-1}B^{(1)}_{n-1,i}\right)F_i = B^{(1)}_{n-1,n-1}g_{j1} - B^{(1)}_{j,n-1}g_{n-1,1} \quad (j=\overline{1,n-2}), \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \sum\limits_{i=1}^{n-2} B^{(1)}_{n-1,i}F_i + B^{(1)}_{n-1,n-1}F_{n-1} = g_{n-1,1}. \end{cases} \tag{11}$$

Introducing the auxiliary notations for the corresponding operators and functions

$$\begin{aligned} &B^{(1)}_{n-1,n-1}B^{(1)}_{ji} - B^{(1)}_{j,n-1}B^{(1)}_{n-1,i} = B^{(2)}_{ji} \quad (j,i=\overline{1,\,n-2}), \\ &B^{(1)}_{n-1,n-1}g_{j1} - B^{(1)}_{j,n-1}g_{n-1,1} = g_{j2} \quad (j=\overline{1,\,n-2}) \end{aligned} \tag{12}$$

we can rewrite now the system (11) without its "single equation":

$$\sum_{i=1}^{n-2} B^{(2)}_{ji}F_i = g_{j2} \quad (j=\overline{1,\,n-2}). \tag{13}$$

Continuing the proposed procedure, at every consequent step we obtain the "subsystem" of the concluding system from the previous algorithm stage, i.e. the former final system without its "single equation". Each of these new studied objects has by one component F_i less than it had at the preceding step. It should be remembered here that since, at each step's closing the "single equation" is rejected hence, we can speak about the equivalence of the obtaining systems only within the limits of the certain algorithmic stage and only until the moment of temporary rejection of the corresponding "single equation". Anyhow, it is obvious and will be completely evident at the end of the present part 2 that the sought for system which is equivalent to the initial system (4), will be obtained after the final step $k = n-1$ by the attachment to the last concluding scalar equation all the preceding "single" non-scalar ones that were rejected before.

Thus, generalizing the given method for any $k=\overline{1,\,n-1}$, we can write the resulting system of the *k*th step

$$\sum_{i=1}^{n-k} B^{(k)}_{ji}F_i = g_{jk} \quad (j=\overline{1,n-k}) \quad (k=\overline{1,n-1}), \tag{14}$$

and consider the general

Step $k+1$ for $\forall\, k=\overline{1,\,n-1}$. In all $n-k$ equations of system (14) we isolate the item with the component F_{n-k}, and the last equation of (14) is written separately:

$$\begin{cases} \sum\limits_{i=1}^{n-k-1} B^{(k)}_{ji}F_i + B^{(k)}_{j,n-k}F_{n-k} = g_{jk} \quad (j=\overline{1,\,n-k-1}), \\ \sum\limits_{i=1}^{n-k-1} B^{(k)}_{n-k,i}F_i + B^{(k)}_{n-k,n-k}F_{n-k} = g_{n-k,k} \quad (k=\overline{1,\,n-1}). \end{cases} \tag{15}$$

Then we apply to the ($n-k$)th equation of (15) the following operator

$$\left(-B^{(k)}_{j,n-k}\right) \quad (j=\overline{1,\,n-k-1}) \tag{15'}$$

consistently for all j from (15′), and to the rest $n-k-1$ equations of the same system the operator

$$B^{(k)}_{n-k,n-k} \tag{15''}$$

is applied. Afterwards we sum in the consecutive order the $(n-k)$th transformed equation and the rest $n-k-1$ transformed equations from system (15) for all $j=\overline{1,n-k-1}$. As the result we come to the following system

$$\begin{cases} \sum\limits_{i=1}^{n-k-1} \left(B^{(k)}_{n-k,n-k} B^{(k)}_{ji} - B^{(k)}_{j,n-k} B^{(k)}_{n-k,i} \right) F_i = B^{(k)}_{n-k,n-k} g_{jk} - B^{(k)}_{j,n-k} g_{n-k,k} \quad (j=\overline{1,n-k-1}), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \sum\limits_{i=1}^{n-k-1} B^{(k)}_{n-k,i} F_i + B^{(k)}_{n-k,n-k} F_{n-k} = g_{n-k,k} \end{cases} \tag{16}$$

that is equivalent to (15). The first $n-k-1$ equations of (16) contain only components $F_i\,(i=\overline{1,n-k-1})$ and have no $F_i\,(i=\overline{n-k,n})$. The $(n-k)$th equation of the system (16) is “single”.

Introducing the auxiliary notations for the appropriate certain operators and functions

$$\begin{gathered} B^{(k)}_{n-k,n-k} B^{(k)}_{ji} - B^{(k)}_{j,n-k} B^{(k)}_{n-k,i} = B^{(k+1)}_{ji} \quad (j,i=\overline{1,n-k-1}), \\ B^{(k)}_{n-k,n-k} g_{jk} - B^{(k)}_{j,n-k} g_{n-k,k} = g_{j,k+1} \quad (j=\overline{1,n-k-1}) \end{gathered} \tag{17}$$

we can write the concluding system of the current step $k+1$, i.e. — (16) without its ”single” equation

$$\sum_{i=1}^{n-k-1} B^{(k+1)}_{ji} F_i = g_{j,k+1} \quad (j=\overline{1,n-k-1}) \quad (k=\overline{1,n-1}). \tag{18}$$

The known operators B... and functions g... from (18) are defined by the formulae (8), (17).

And at last the final

Step k = $n-1$ leads to the following: we substitute $k+1=n-1 \Leftrightarrow k=n-2$ to (17), (18) and as the result obtain the required scalar equation with the component F_1

$$B^{(n-1)}_{11} F_1 = g_{1,n-1}, \tag{19}$$

while the rest $n-1$ nonscalar equations are ”single”. In (19) the corresponding given operator and function are described by the below written recurrent formulae that were obtained after the substitution of $k=n-2$ for (17):

$$\begin{gathered} B^{(n-1)}_{11} = B^{(n-2)}_{22} B^{(n-2)}_{ji} - B^{(n-2)}_{j2} B^{(n-2)}_{2i} \quad (j,i=1), \\ g_{1,n-1} = B^{(n-2)}_{22} g_{1,n-2} - B^{(n-2)}_{12} g_{2,n-2}. \end{gathered} \tag{20}$$

Here it should be noted that in (17), (18**),** (20) as everywhere in the present part 2, the upper index in the round brackets of the known operator B... and the second lower index of the given function g... mean the step number of the diagonalization procedure in the “upward” direction.

Also we have to notice that even at the current stage of the only one scalar equation’s construction we need extremely the operator commutativity in pairs (5). This evident fact follows directly from the realization of the proposed algorithm. It is quite enough to agree that at each kth step for all $k=\overline{1,n-1}$ the corresponding operators are applied to the studied equations either only from the left or only from the right side during the whole ”upward” diagonalization process as it is done everywhere in the present part 2.

At last, closing the part 2 which main purpose was attained in the formulae (19). (20), we can write below the final system of the diagonalization procedure in “upward” direction

$$\begin{cases} B^{(n-1)}_{11} F_1 = g_{1,n-1}, & (21) \\ \sum\limits_{i=1}^{n-k} B^{(k)}_{ji} F_1 = g_{jk} \quad (j=\overline{2,n-k}; \quad k=\overleftarrow{0,n-2}), & (*) \end{cases}$$

where:

$$B^{(0)}_{ni} = A_{ni}\ (i=\overline{1,n}), \quad g_{n0} = f_n, \quad \text{when } k=0, \tag{21′}$$

are the known appropriate operators and functions from the last equation of the system (6) or (7). System (21), (*) is obtained by the attachment to the desired scalar equation (19) of all “single” equations that were rejected earlier. Therefore, the equivalence of (21), (*) to the initial system (4) ≡ (6) is obvious.

Additionally, it should be noted that the arrow direction for the index k from formula (*) till the very end of the next part 3 will describe the backward counting, — from the right to the left.

System (21), (*) represents the completion of the ”upward” diagonalization stage.

3. Construction of the rest scalar equations with the components F_i $(i=\overline{2,n})$ (diagonalization "from the top to the bottom"). Now we are going to propose the second diagonalization stage that works in the opposite downward direction, "from the top to the bottom".

Step 1 (k = $n-2$). We isolate the first equation of the subsystem (*) and write it together with the obtained scalar equation (21) that has the component F_1. At this moment we neglect the rest $k=\overleftarrow{0,n-3}$ equations from (*) considering them as "single":

$$\begin{cases} B_{11}^{(n-1)}F_1 = g_{1,n-1}, \\ \sum_{i=1}^{2} B_{ji}^{(n-2)}F_i = g_{j,n-2} \quad (j=2). \end{cases} \tag{22}$$

In the last equation of (22) we separate the item with the scalar F_2

$$\begin{cases} B_{11}^{(n-1)}F_1 = g_{1,n-1}, \\ B_{21}^{(n-2)}F_1 + B_{22}^{(n-2)}F_2 = g_{2,n-2} \end{cases} \tag{23}$$

apply to the second and first equations from (23) the appropriate operators

$$B_{11}^{(n-1)}, \tag{23$'$}$$

$$\left(-B_{21}^{(n-2)}\right) \tag{23$''$}$$

and sum up the both transformed equations.

Turning to the system (23), we may notice that after its recent transformation that dealt with the operators (23′**)**, (23″), we come to the following system

$$\begin{cases} B_{11}^{(n-1)}F_1 = g_{1,n-1}, \\ B_{11}^{(n-1)}B_{22}^{(n-2)}F_2 = B_{11}^{(n-1)}g_{2,n-2} - B_{21}^{(n-2)}g_{1,n-1} \end{cases} \tag{24}$$

which is equivalent to (23) and whose second equation is scalar with respect to the component F_2.

Introducing the auxiliary notation for the known function from the right part of the last equation in system (24)

$$B_{11}^{(n-1)}g_{2,n-2} - B_{21}^{(n-2)}g_{1,n-1} = h_1, \tag{25}$$

we rewrite (24) as follows

$$\begin{cases} B_{11}^{(n-1)}F_1 = g_{1,n-1}, \\ B_{11}^{(n-1)}B_{22}^{(n-2)}F_2 = h_1. \end{cases} \tag{26}$$

It is clear that after getting formulae (26), the subsystem (*) has lessened by one equation and looks like

$$\sum_{i=1}^{n-k} B_{ji}^{(k)}F_i = g_{jk} \quad (j=\overline{3,n-k};\quad k=\overleftarrow{0,n-3}). \tag{$*_1$}$$

Step 2 (k = $n-3$). We isolate now the first equation from system ($*_1$) and attach it to the concluding system of scalar equations (26) from the preceding step 1

$$\begin{cases} B_{11}^{(n-1)}F_1 = g_{1,n-1}, \\ B_{11}^{(n-1)}B_{22}^{(n-2)}F_2 = h_1, \\ \sum_{i=1}^{3} B_{ji}^{(n-3)}F_i = g_{j,n-3} \quad (j=3). \end{cases} \tag{27}$$

The rest $k=\overleftarrow{0,n-4}$ equations in ($*_1$) are "single".

Then in the third equation of (27) we isolate the item with the scalar F_2

$$\begin{cases} B_{11}^{(n-1)} F_1 = g_{1,n-1}, \\ B_{11}^{(n-1)} B_{22}^{(n-2)} F_2 = h_1, \\ \sum\limits_{i=1}^{2} B_{3i}^{(n-3)} F_i + B_{33}^{(n-3)} F_3 = g_{3,n-3}. \end{cases} \tag{28}$$

Afterwards, we apply to the third, first and second equations of (28) the following operators correspondingly

$$B_{11}^{(n-1)} B_{22}^{(n-2)}, \tag{28′}$$

$$\left(-B_{31}^{(n-3)} B_{22}^{(n-2)}\right) \text{ and } \left(-B_{32}^{(n-3)}\right). \tag{28″}$$

Summing up these three transformed equations we get the system

$$\begin{cases} B_{11}^{(n-1)} F_1 = g_{1,n-1}, \\ B_{11}^{(n-1)} B_{22}^{(n-2)} F_2 = h_1, \\ B_{11}^{(n-1)} B_{22}^{(n-2)} B_{33}^{(n-3)} F_3 = B_{11}^{(n-1)} B_{22}^{(n-2)} g_{3,n-3} - \left(B_{31}^{(n-3)} B_{22}^{(n-2)} g_{1,n-1} + B_{32}^{(n-3)} h_1\right) \end{cases} \tag{29}$$

that is equivalent to (28) and has already the third required scalar equation with the component F_3.

Introducing the auxiliary notation for the known function from the right part of the last equation in system (29)

$$B_{11}^{(n-1)} B_{22}^{(n-2)} g_{3,n-3} - \left(B_{31}^{(n-3)} B_{22}^{(n-2)} g_{1,n-1} + B_{32}^{(n-3)} h_1\right) = h_2, \tag{30}$$

we rewrite (29) as follows

$$\begin{cases} B_{11}^{(n-1)} F_1 = g_{1,n-1}, \\ B_{11}^{(n-1)} B_{22}^{(n-2)} F_2 = h_1, \\ B_{11}^{(n-1)} B_{22}^{(n-2)} B_{33}^{(n-3)} F_3 = h_2. \end{cases} \tag{31}$$

Indices of the given functions h... from (25), (30) and everywhere in the present section 3 imply the step number of the second diagonalization stage “from the top to the bottom”.

The subsystem ($*_1$) decreases now by one equation more (the original subsystem (*) — respectively by two) and turns into the following

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} F_i = g_{jk} \quad (j = \overline{4, n-k}; \quad k = \overleftarrow{0, n-4}). \tag{$*_2$}$$

Further, the generalization of two preceding moments of the current diagonalization stage in the case of the arbitrary step $l\ (l = \overline{1, n-1})$ is considered. At first we rewrite the subsystem $(*_{l-1})$ of the previous step $l-1$

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} F_i = g_{jk} \quad (j = \overline{l+1, n-k}; \quad k = \overleftarrow{0, n-l-1}). \tag{$*_{l-1}$}$$

When $l = 1$, the second equation in (22) corresponds to the ”zero” step.

As it was done earlier in the section 3, we isolate the first equation in $(*_{l-1})$ and attach it to the concluding system of scalar equations from the preceding step $l = 1$. Simultaneously, the remained $k = \overleftarrow{0, n-l-2}$ equations in $(*_{l-1})$ are “single”.

The system which last equation will be reduced to a scalar one is obtained earlier and looks like

$$\begin{cases} \prod\limits_{q=1}^{p+1} B_{qq}^{(n-q)} F_{p+1} = h_p \quad (p = \overline{0, l-1};\ h_0 = g_{1,n-1}), \\ \sum\limits_{i=1}^{l+1} B_{ji}^{(n-l-1)} F_i = g_{j,n-l-1} \quad (j = l+1). \end{cases} \tag{32}$$

The symbol of the finite operator product in (32) and later in the present part 3 implies the usual consequent operator application from the inner to the external in “the right to the left” direction.

Further, we separate in the $(l+1)$th equation of (32) the item with the component F_{l+1}

$$\begin{cases} \prod_{q=1}^{p+1} B_{qq}^{(n-q)} F_{p+1} = h_p \quad (p = \overline{0, l-1}; h_0 = g_{1,n-1}), \\ \sum_{i=1}^{l} B_{l+1,i}^{(n-l-1)} F_i = g_{l+1,n-l-1} \end{cases} \tag{33}$$

and apply to the last equation from (33) the operator

$$\prod_{q=1}^{l} B_{qq}^{(n-q)}. \tag{33′}$$

To the remained equations in (33) from the first till the $(l-1)$th we apply the appropriate operators

$$\left(-B_{l+1,r}^{(n-l-1)} \prod_{q=r+1}^{l} B_{qq}^{(n-q)} \right) \quad (r = \overline{1, l-1}) \tag{33″}$$

and the lth equation of the same system is transformed by the operator

$$\left(-B_{l+1,l}^{(n-l-1)} \right). \tag{33‴}$$

Then we sum up all these $l+1$ transformed equations and obtain the system

$$\begin{cases} \prod_{q=1}^{p+1} B_{qq}^{(n-q)} F_{p+1} = h_p \quad (p = \overline{0, l-1}; h_0 = g_{1,n-1}), \\ \prod_{q=1}^{l+1} B_{qq}^{(n-q)} F_{l+1} = \prod_{q=1}^{l} B_{qq}^{(n-q)} g_{l+1,n-l-1} - \sum_{\substack{r=1 \\ (l \neq 1)}}^{l-1} B_{l+1,r}^{(n-l-1)} \prod_{q=r+1}^{l} B_{qq}^{(n-q)} h_{r-1} - B_{l+1,l}^{(n-l-1)} h_{l-1} \end{cases} \tag{34}$$

that is equivalent to (33). In the case of $l=1$ the second item in the right part of the last equation from (34) is assumed to be equal to zero.

Introducing the common notation for the known function from the right part of the last equation in (34)

$$h_l = \prod_{q=1}^{l} B_{qq}^{(n-q)} g_{l+1,n-l-1} - \sum_{\substack{r=1 \\ (l \neq 1)}}^{l-1} B_{l+1,r}^{(n-l-1)} \prod_{q=r+1}^{l} B_{qq}^{(n-q)} h_{r-1} - B_{l+1,l}^{(n-l-1)} h_{l-1} \tag{35}$$

we can write the final system (34) of the arbitrary step $l \ (l = \overline{1, n-2})$ as follows

$$\prod_{q=1}^{p+1} B_{qq}^{(n-q)} F_{p+1} = h_p \quad (p = \overline{0, l}; \ h_0 = g_{1,n-1}) \tag{36}$$

and the second item from the right part of (35) is equal to zero when $l=1$.

The obtained recurrent formulae (35), (36) are easily verified, e.g. for the above mentioned steps $l = 1, 2$.

In should be noted that after the construction of (36) the subsystem $(*_{l-1})$ decreases by one equation (the initial subsystem (*) — correspondingly by l) and turns into the following

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} F_i = g_{ik} \quad (j = \overline{l+2, n-k}; \quad k = \overleftarrow{0, n-l-2}). \tag{$*_l$}$$

After continuation of the second diagonalization stage in the downward direction including the final step $l = n-1$, we come to the sought for system of the scalar equations with all components $F_i (i = \overline{1, n})$ (look (36) when $l = n-1$):

$$\prod_{q=1}^{p+1} B_{qq}^{(n-q)} F_{p+1} = h_p \quad (p = \overline{0, n-1}; \ h_0 = g_{1,n-1}) \tag{37}$$

where the certain operators and functions are described by the formulae (17) and (35) from the parts 2 and 3 correspondingly.

When the explicit construction of the resulting system (37) is finished, we can assert that the subsystem of “single”equations $(*_l)$ does not exist, since after completion of the preceding step $l = n-2$ the subsystem $(*_l)$ consisted of one equation

$$\sum_{i=1}^{n-k} B_{ji}^{(k)} F_i = g_{ik} \quad (j = \overline{n, n-k}; \quad k = 0)$$

$$\Updownarrow$$

$$\sum_{i=1}^{n} B_{ni}^{(0)} F_i = g_{n0}. \qquad (*_{n-2})$$

In $(*_{n-2})$ the given operator B... and function g... are from the formulae (21′). The next final step $l = n-1$ brings $(*_{n-2})$ to the sought for scalar equation with the component F_n.

At the end of the present section 3 we must mark the equivalence of the wanted scalar equations' system (37) and the initial system (21), (*). Therefore, the both mentioned systems are equivalent to the original system (4). This fact follows directly from the proposed diagonalization procedure and completes it. Thus, the existence of the initial operator system solution is proved in the diagonalization terms and the main purpose of the given paper is attained.

4. Algorithm application to the matrix block operators in the case of the differential Maxwell system. From the previous parts 2, 3 we can easily conclude that the proposed diagonalization procedure is invariant concerning the matrix construction of the initial system (4), i.e. the operators $A_{ji}(j, i = \overline{1, n})$ which satisfy (5) may have the arbitrary block structure. In this case the original matrix is diagonalized at first, and its every block is considered as one operator. When the desired diagonalized block-matrix is formed, the consecutive diagonalization of each block follows and, as the result the sought for scalar equations are finally obtained.

As the example we apply the proposed algorithm to the classical Maxwell system (3) that may be written in the following block way [8]:

$$\begin{bmatrix} I_1 & \partial_+ \\ -\partial_+ & I_2 \end{bmatrix} \begin{bmatrix} E \\ H \end{bmatrix} = \begin{bmatrix} j^{\text{ст}} \\ e^{\text{ст}} \end{bmatrix}, \qquad (38)$$

where: $E, H, j^{\text{ст}}$ are from the part 1 and in our case $e^{\text{ст}} \equiv 0$.

$$\partial_+ = \begin{bmatrix} 0 & -\partial_3 & \partial_2 \\ \partial_3 & 0 & -\partial_1 \\ -\partial_2 & \partial_1 & 0 \end{bmatrix}, \quad I_1 = -(\sigma + \varepsilon_a \partial_0) I, \quad I_2 = -\mu_a \partial_0 I, \qquad (39)$$

I is the 3-dimensional unit matrix; operator notations $\partial_i = (i = \overline{1, 3})$ $\left(\partial_1 = \frac{\partial}{\partial x}, \partial_2 = \frac{\partial}{\partial y}, \partial_3 = \frac{\partial}{\partial z}\right)$ and constants $\sigma, \varepsilon_a, \mu_a$ remain the same as they are in the part 1.

The diagonalization of the system (38) is done according to the procedure of the previous sections 2, 3. From the very beginning we consider each block of (38) as one operator and write (38) in the equivalent form

$$\begin{cases} I_1 E + \partial_+ H = j^{\text{ст}}, \\ -\partial_+ E + I_2 H = e^{\text{ст}}. \end{cases} \qquad (40)$$

After application of the appropriate operators

$$I_2 \text{ and } (-\partial_+)$$

to the first and second equations of (40) respectively and summing up these transformed equations, we come to the system

$$\begin{cases} \left(I_1 I_2 + \partial_+^2\right) E = I_2 j^{\text{ст}} - \partial_+ e^{\text{ст}}, \\ -\partial_+ E + I_2 H = e^{\text{ст}} \end{cases} \qquad (41)$$

that is equivalent to (40) and (38). The first equation in (41) has the only one unknown function E which is the vector one yet. Hence, we can assert that the final result of the part 2 in block terms is obtained.

Further, we propose the second stage of the described algorithm from the part 3. It means that the following operators

$$I_1 I_2 + \partial_+^2 \text{ and } \partial_+$$

are applied to the second and first equations of (41) respectively. Afterwards, these two transformed equations are summed up and the system which is equivalent to (41) is obtained

$$\begin{cases} \left(I_1 I_2 + \partial_+^2\right) E = I_2 j^{\text{ст}} - \partial_+ e^{\text{ст}}, \\ \left(I_1 I_2 + \partial_+^2\right) I_2 H = \left(I_1 I_2 + \partial_+^2\right) e^{\text{ст}} + \partial_+ \left(I_2 j^{\text{ст}} - \partial_+ e^{\text{ст}}\right). \end{cases}$$

After obvious calculations the last system looks like

$$\begin{cases} \left(I_1 I_2 + \partial_+^2\right) E = I_2 j^{\text{ст}} - \partial_+ e^{\text{ст}}, \\ \left(I_1 I_2 + \partial_+^2\right) I_2 H = I_2 \left(\partial_+ j^{\text{ст}} + I_1 e^{\text{ст}}\right) \end{cases} \tag{42}$$

and its second equation has the only one function H that is also the vector one yet. Therefore, the final result of the part 3 in block terms is also obtained and the diagonalization process of system (3) at the vector level is finished.

Now we begin the separate diagonalization procedure of the first and second vector equations from (42) with respect to the corresponding scalar components $E = \{E_i\}_{i=1}^3$ and $H = \{H_i\}_{i=1}^3$.

From the very beginning we have to calculate the following operator block-matrices, taking into account notations (39)

$$\partial_+^2 = \begin{bmatrix} -\partial_3^2 - \partial_2^2 & \partial_1 \partial_2 & \partial_1 \partial_3 \\ \partial_1 \partial_2 & -\partial_3^2 - \partial_1^2 & \partial_2 \partial_3 \\ \partial_1 \partial_3 & \partial_2 \partial_3 & -\partial_2^2 - \partial_1^2 \end{bmatrix}, \quad I_3 = I_1 I_2 = \mu_a \partial_0 (\sigma + \varepsilon_a \partial_0) I. \tag{43}$$

Putting the below written expressions

$$\begin{gathered} \hat{\partial}_0^* = \mu_a \partial_0 (\sigma + \varepsilon_a \partial_0), \\ \Delta = \partial_1^2 + \partial_2^2 + \partial_3^2 \end{gathered} \tag{44}$$

into the formulae (43), we get the operator matrices

$$\partial_+^2 = \begin{bmatrix} \partial_1^2 - \Delta & \partial_1 \partial_2 & \partial_1 \partial_3 \\ \partial_1 \partial_2 & \partial_2^2 - \Delta & \partial_2 \partial_3 \\ \partial_1 \partial_2 & \partial_2 \partial_3 & \partial_3^2 - \Delta \end{bmatrix}, \quad I_3 = \hat{\partial}_0^* I. \tag{45}$$

From (43) – (45) we conclude that

$$\partial_+^2 + I_3 = \begin{bmatrix} \hat{\partial}_0^* + \partial_1^2 - \Delta & \partial_1 \partial_2 & \partial_1 \partial_3 \\ \partial_1 \partial_2 & \hat{\partial}_0^* + \partial_2^2 - \Delta & \partial_2 \partial_3 \\ \partial_1 \partial_3 & \partial_2 \partial_3 & \hat{\partial}_0^* + \partial_3^2 - \Delta \end{bmatrix}. \tag{46}$$

Hence, the first block equation of (42) that is scalar with respect to the vector E, in its coordinate form may be written as:

$$\sum_{i=1}^{3} A_{ji} F_i = f_j \quad (j = \overline{1,3}), \tag{47}$$

where:

$$\begin{gathered} F_i = E_i, \quad f_j = -\mu_a \partial_0 j_j^{\text{ст}} \quad (j, i = \overline{1,3}), \\ A_{ji} = A_{ij} = \partial_j \partial_i \quad \text{when} \quad i \neq j \quad (j, i = \overline{1,3}), \\ A_{jj} = A_{ii} = \hat{\partial}_0^* + \partial_j^2 - \Delta \quad \text{when} \quad i = j \quad (j, i = \overline{1,3}). \end{gathered} \tag{48}$$

Then we apply to (47) the first diagonalization stage from the section 2 and use the formulae (17), (19), (20) when $k=\overline{1,n-1}$; $n=3$. The last approach leads to the required scalar equation that has the first component of the unknown vector-function $\vec{E}$:

$$B_{11}^{(2)}E_1=g_{12}, \tag{49}$$

and

$$g_{12}=B_{22}^{(1)}g_{11}-B_{12}^{(1)}g_{21}. \tag{49′}$$

After application to (49′) the equality (8) from the part 2, we can rewrite the given operators and functions of formula (49′) as follows:

$$\begin{aligned} &B_{ji}^{(1)}=A_{33}A_{ji}-A_{j3}A_{3i} \quad (j,i=1,2),\\ &g_{i1}=A_{33}f_j-A_{j3}f_3 \quad (j=1,2). \end{aligned} \tag{49″}$$

Since in the considered particular case the first diagonalization stage for $\vec{E}$ and $\vec{H}$ has only two steps, then for the left part of (49) it is sufficient to use the first equality (12) from the preceding section 2:

$$B_{11}^{(2)}=B_{22}^{(1)}B_{11}^{(1)}-B_{12}^{(1)}B_{21}^{(1)}. \tag{49‴}$$

Taking into account formula (49″) and symmetry of the operators A_{ji} with respect to the main diagonal of the matrix (46), we can simplify the “right-side” operators from (49‴)

$$B_{jj}^{(1)}=A_{33}A_{jj}-A_{j3}^{(2)} \quad (j=i=\overline{1,2}),$$

$$B_{21}^{(1)}=B_{12}^{(1)}=A_{33}A_{12}-A_{13}A_{32}.$$

Putting into the last formulae the operator explicit values from (48) and using evident identities

$$(\hat{\partial}_0^*+\partial_3^2-\Delta)\,(\hat{\partial}_0^*+\partial_j^2-\Delta)-\partial_j^2\partial_3^2=(\hat{\partial}_0^*-\Delta)^2+(\hat{\partial}_0^*-\Delta)\,(\partial_3^2+\partial_j^2)=(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\partial_i^2),\quad (i\neq j)$$

$$(\hat{\partial}_0^*+\partial_3^2-\Delta)\,\partial_1\partial_2-\partial_1\partial_3^2\partial_2=\partial_1\partial_2(\hat{\partial}_0^*-\Delta),$$

we get

$$\begin{aligned} &B_{jj}^{(1)}=(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\partial_i^2) \quad (i\neq j;\ i,j=1,2),\\ &B_{21}^{(1)}=B_{12}^{(1)}=\partial_1\partial_2(\hat{\partial}_0^*-\Delta). \end{aligned} \tag{50}$$

Then the right part of (49‴) by means of (50) comes to its final operator expression

$$B_{11}^{(2)}=(\hat{\partial}_0^*-\Delta)^2\,(\hat{\partial}_0^*-\partial_2^2)\,(\hat{\partial}_0^*-\partial_1^2)-\partial_1^2\partial_2^2(\hat{\partial}_0^*-\Delta)^2,$$

which after simple calculations

$$(\hat{\partial}_0^*-\Delta)^2\,(\hat{\partial}_0^*-\partial_2^2)\,(\hat{\partial}_0^*-\partial_1^2)-\partial_1\partial_2(\hat{\partial}_0^*-\Delta)^2=(\hat{\partial}_0^*-\Delta)^2\left(\hat{\partial}_0^{*2}-\hat{\partial}_0^*(\partial_1^2+\partial_2^2)\right)=(\hat{\partial}_0^*-\Delta)^2\,\hat{\partial}_0^*(\hat{\partial}_0^*-\Delta+\partial_3^2)$$

and the second notation of (44) may be written as follows

$$B_{11}^{(2)}=(\hat{\partial}_0^*-\Delta)^2\,\hat{\partial}_0^*(\hat{\partial}_0^*-\Delta+\partial_3^2). \tag{51}$$

Further, we define the explicit construction of the function from formula (49′) Approaching this direction we substitute (49″) for (49′) and after simple transformations using the operators’ A_{ji} $(j,i=\overline{1,n})$ commutativity, can obtain

$$g_{12}=A_{33}\left(B_{22}^{(1)}f_1-B_{12}^{(1)}f_2\right)+\left(B_{12}^{(1)}A_{23}-B_{22}^{(1)}A_{13}\right)f_3.$$

If we write the last formula in terms of (50), (48)

$$\begin{aligned} g_{12}=&(\hat{\partial}_0^*-\Delta+\partial_3^2)\left((\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\partial_1^2)\,(-\mu_a\partial_0 j_1^{\text{ст}})-\partial_1\partial_2(\hat{\partial}_0^*-\Delta)(-\mu_a\partial_0 j_2^{\text{ст}})\right)+\\ &+\left(\partial_1\partial_2(\hat{\partial}_0^*-\Delta)\partial_2\partial_3-(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\partial_1^2)\partial_1\partial_3\right)(-\mu_a\partial_0 j_3^{\text{ст}}) \end{aligned}$$

and transform it by the second equality of (44)

$$\begin{aligned} &-\mu_a\partial_0(\hat{\partial}_0^*-\Delta)\left((\hat{\partial}_0^*-\Delta+\partial_3^2)\left((\hat{\partial}_0^*-\partial_1^2)j_1^{\text{ст}}-\partial_1\partial_2 j_2^{\text{ст}}\right)+\partial_1\partial_3(\partial_2^2-\hat{\partial}_0^*+\partial_1^2)j_3^{\text{ст}}\right)=\\ &=-\mu_a\partial_0(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\Delta+\partial_3^2)\left((\hat{\partial}_0^*-\partial_1^2)j_i^{\text{ст}}-\partial_1(\partial_2 j_2^{\text{ст}}+\partial_3 j_3^{\text{ст}})\right), \end{aligned}$$

then we can submit the concluding formula for g_{12}

$$g_{12}=-\mu_a\partial_0(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\Delta+\partial_3^2)\left((\hat{\partial}_0^*-\partial_1^2)\,j_1^{\text{ст}}-\partial_1(\partial_2 j_2^{\text{ст}}+\partial_3 j_3^{\text{ст}})\right). \tag{52}$$

At last, putting (51), (52) into the left and right parts of (49) respectively we get

$$(\hat{\partial}_0^*-\Delta)^2\,\hat{\partial}_0^*(\hat{\partial}_0^*-\Delta+\partial_3^2)E_1=-\mu_a\partial_0(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\Delta+\partial_3^2)\left((\hat{\partial}_0^*-\partial_1^2)\,j_1^{\text{ст}}-\partial_1(\partial_2 j_2^{\text{ст}}+\partial_3 j_3^{\text{ст}})\right).$$

Neglecting the “common operator multiplier”

$$-(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\Delta+\partial_3^2)\;\mu_a\partial_0 \tag{53}$$

and taking into account the first equality (44), we can write the sought for scalar equation with respect to the component E_1 of the initial vector-function $\vec{E}$ [10]

$$(\sigma+\varepsilon_a\partial_0)\,(\hat{\partial}_0^*-\Delta)\,E_1=(\partial_1^2-\hat{\partial}_0^*)\,j_1^{\text{ст}}+\partial_1(\partial_2 j_2^{\text{ст}}+\partial_3 j_3^{\text{ст}}). \tag{54}$$

Further, we use the second diagonalization stage of the part 3 when p = 1, 2 in the formulae (37), (35). As the result we obtain the sought for scalar equations with the components E_2, E_3

$$\begin{gathered}\prod_{q=1}^{2}B_{qq}^{(3-q)}E_2=h_1,\\ \prod_{q=1}^{3}B_{qq}^{(3-q)}E_3=h_2,\end{gathered} \tag{55}$$

where $B_{11}^{(2)}, B_{22}^{(2)}$ are defined by the corresponding formulae (51), (50) of the present section 4,

$$B_{33}^{(0)}=A_{33}=\hat{\partial}_0^*+\partial_3^2-\Delta \tag{56}$$

is written according to (21′) from the part 2 and $(*_{n-2})$ is from the part 3. Taking into account (55) we can write functions (35) as follows

$$\begin{gathered}h_1=B_{11}^{(2)}g_{21}-B_{21}^{(1)}h_0,\quad h_0=g_{21}\quad\text{by (36)},\\ h_2=B_{22}^{(1)}\left(B_{11}^{(2)}g_{30}-B_{31}^{(0)}h_0\right)-B_{32}^{(0)}h_1,\end{gathered} \tag{57}$$

$$B_{3i}^{(0)}=A_{3i}\quad(i=\overline{1,3}),\quad g_{30}=f_3\ \text{by (21′)}. \tag{58}$$

Putting expressions (48) into (57), (58), due to the formulae (50), (51), (52), (49″), (56) we can propose the sought for scalar equations (55) in the explicit form

$$\begin{aligned}&(\hat{\partial}_0^*-\Delta)^3\,(\hat{\partial}_0^*-\partial_1^2)\,\hat{\partial}_0^*(\hat{\partial}_0^*-\Delta+\partial_3^2)\,E_2=\\ &=(\hat{\partial}_0^*-\Delta)^2\,\hat{\partial}_0^*(\hat{\partial}_0^*-\Delta+\partial_3^2)\left((\hat{\partial}_0^*+\partial_3^2-\Delta)\,(-\mu_a\partial_0 j_2^{\text{ст}})-\partial_2\partial_3(-\mu_a\partial_0 j_3^{\text{ст}})\right)+\\ &+\partial_1\partial_2\,(\hat{\partial}_0^*-\Delta)\,\mu_a\partial_0\,(\hat{\partial}_0^*-\Delta)\,(\hat{\partial}_0^*-\Delta+\partial_3^2)\left((\hat{\partial}_0^*-\partial_1^2)\,j_1^{\text{ст}}-\partial_1(\partial_2 j_2^{\text{ст}}+\partial_3 j_3^{\text{ст}})\right);\\ &(\hat{\partial}_0^*-\Delta)^3\,\hat{\partial}_0^*(\hat{\partial}_0^*-\Delta+\partial_3^2)\,(\hat{\partial}_0^*-\partial_1^2)E_3=-\mu_a\partial_0\,(\hat{\partial}_0^*-\Delta)^2\,(\hat{\partial}_0^*-\partial_1^2)\,(\hat{\partial}_0^*+\partial_3^2-\Delta)\cdot\\ &\cdot\left((\hat{\partial}_0^*-\Delta)\hat{\partial}_0^*\,j_3^{\text{ст}}+\partial_1\partial_3\left(\partial_1(\partial_2 j_2^{\text{ст}}+\partial_3 j_3^{\text{ст}})-(\hat{\partial}_0^*-\partial_1^2)\,j_1^{\text{ст}}\right)\right)-\partial_3\partial_2 h_1,\end{aligned} \tag{59}$$

where h_1 is the right part of the first equation in (59).

Further, we transform the right part h_1 of the first equation from (59) as follows

$$h_1=(\hat{\partial}_0^*-\Delta)^2\,(\hat{\partial}_0^*-\Delta+\partial_3^2)\;\mu_a\partial_0\left((\hat{\partial}_0^*-\partial_1^2)\,\partial_2(\partial_1 j_1^{\text{ст}}+\partial_3 j_3^{\text{ст}})-\left(\hat{\partial}_0^*(\hat{\partial}_0^*+\partial_3^2-\Delta)+\partial_1^2\partial_2^2\right)j_2^{\text{ст}}\right).$$

After application to the last expression the operator identity

$$\hat{\partial}_0^*(\hat{\partial}_0^*+\partial_3^2-\Delta)+\partial_1^2\partial_2^2=\hat{\partial}_0^*(\hat{\partial}_0^*-\partial_1^2-\partial_2^2)+\partial_1^2\partial_2^2=\hat{\partial}_0^*(\hat{\partial}_0^*-\partial_1^2)-\partial_2^2(\hat{\partial}_0^*-\partial_1^2)=(\hat{\partial}_0^*-\partial_1^2)\,(\hat{\partial}_0^*-\partial_2^2),$$

the known function h_1 looks like

$$h_1=(\hat{\partial}_0^*-\Delta)^2\,(\hat{\partial}_0^*-\Delta+\partial_3^2)\;\mu_a\partial_0(\hat{\partial}_0^*-\partial_1^2)\left((\partial_2^2-\hat{\partial}_0^*)\;j_2^{\text{ст}}+\partial_2(\partial_1 j_1^{\text{ст}}+\partial_3 j_3^{\text{ст}})\right). \tag{60}$$

Putting (60) into the right part of the first equation in (59) and neglecting the “common operator multiplier”

$$(\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,(\hat{\partial}_0^* - \partial_1^2)\,\mu_a \partial_0, \tag{61}$$

we come to the final scalar equation with respect to the component E_2 of the unknown vector-function $\vec{E}$ [10]

$$(\hat{\partial}_0^* - \Delta)\,(\sigma + \varepsilon_a \partial_0)\,E_2 = (\partial_2^2 - \hat{\partial}_0^*)\,j_2^{\text{ст}} + \partial_2(\partial_1 j_1^{\text{ст}} + \partial_3 j_3^{\text{ст}}). \tag{62}$$

Then we put (60) to the right part h_2 of the second equation from (59):

$$h_2 = -\mu_a \partial_0 (\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \partial_1^2)\,(\hat{\partial}_0^* + \partial_3^2 - \Delta)\left(\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta) j_3^{\text{ст}} + \partial_1\partial_3\left(\partial_1(\partial_2 j_2^{\text{ст}} + \partial_3 j_3^{\text{ст}}) - (\hat{\partial}_0^* - \partial_1^2) j_1^{\text{ст}}\right)\right) -$$

$$-\partial_3\partial_2(\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\mu_a \partial_0 (\hat{\partial}_0^* - \partial_1^2)\left((\partial_2^2 - \hat{\partial}_0^*) j_2^{\text{ст}} + \partial_2(\partial_1 j_1^{\text{ст}} + \partial_3 j_3^{\text{ст}})\right),$$

and after simple operator calculations bring the last formula to the equivalent form

$$\begin{aligned} h_2 = -\mu_a \partial_0 (\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,(\hat{\partial}_0^* - \partial_1^2)\Big(\left(\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta) + \partial_3^2(\partial_1^2 + \partial_2^2)\right) j_3^{\text{ст}} - \\ -(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\partial_3(\partial_1 j_1^{\text{ст}} + \partial_2 j_2^{\text{ст}})\Big). \end{aligned} \tag{63}$$

Application of the operator identity

$$\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta) + \partial_3^2(\partial_1^2 + \partial_2^2) = (\hat{\partial}_0^* - \partial_3^2)\,(\hat{\partial}_0^* - \Delta + \partial_3^2)$$

to the formula (63) leads to the final representation of the function h_2

$$h_2 = \mu_a \partial_0 (\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)^2\,(\hat{\partial}_0^* - \partial_1^2)\left((\partial_3^2 - \hat{\partial}_0^*) j_3^{\text{ст}} + \partial_3(\partial_1 j_1^{\text{ст}} + \partial_2 j_2^{\text{ст}})\right). \tag{64}$$

Putting (64) into the right part of the second equation from (59) and neglecting the “common operator multiplier”

$$(\hat{\partial}_0^* - \Delta)^2\,\mu_a \partial_0 (\hat{\partial}_0^* - \Delta + \partial_3^2)^2\,(\hat{\partial}_0^* - \partial_1^2), \tag{65}$$

we come to the unknown scalar equation with the last component E_3 of the initial vector-function $\vec{E}$ [10]

$$(\sigma + \varepsilon_a \partial_0)\,(\hat{\partial}_0^* - \Delta)\,E_3 = (\partial_3^2 - \hat{\partial}_0^*)\,j_3^{\text{ст}} + \partial_3(\partial_1 j_1^{\text{ст}} + \partial_2 j_2^{\text{ст}}). \tag{66}$$

Comparing the explicit final equations (54), (62) and (66), we can write the general scalar equation that has simultaneously all components E_i $(i = \overline{1,3})$ of the initial vector-function $\vec{E}$ which describes the electric field tension

$$(\sigma + \varepsilon_a \partial_0)\,(\hat{\partial}_0^* - \Delta)\,E_i = (\partial_i^2 - \hat{\partial}_0^*)\,j_i^{\text{ст}} + \partial_i(\partial_\nu j_\nu^{\text{ст}} + \partial_\lambda j_\lambda^{\text{ст}}) \quad (i = \overline{1,3};\ \ \nu, \lambda \ne i;\ \ \nu \ne \lambda). \tag{67}$$

Formula (67) coincides completely with the particular result of [10].

Further, we diagonalize the second vector equation (42) that depends on the components $\{H_i\}_{i=1}^3$ of the unknown vector-function $\vec{H}$ which describes the magnetic field tension. Taking into account $e^{\text{ст}} \equiv 0$ [10] and neglecting the ”common operator multiplier” I_2 we write the above mentioned equation in the equivalent form

$$\left(I_1 I_2 + \partial_+^2\right) H = \partial + j^{\text{ст}}, \tag{68}$$

$$\Updownarrow$$

$$\sum_{i=1}^{3} A_{ji} H_i = \varphi_j \quad (j = \overline{1,3}), \tag{69}$$

where: A_{ji} are from (48), since the left-side operators of (68) together with the operators of the first equation from (42) are completely identical; vector

$$\varphi = \partial_+ j^{\text{ст}} \Leftrightarrow \varphi_j = \begin{bmatrix} -\partial_3 j_2^{\text{ст}} + \partial_2 j_3^{\text{ст}} \\ \partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}} \\ -\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}} \end{bmatrix} \quad (j = \overline{1,3}). \tag{70}$$

According to the formula (39). Thus, the systems (69), (47) are exactly identical in terms of the $H_i = F_i \quad (i=\overline{1,3}),\ \varphi_j = f_j \quad (j=\overline{1,3})$, where the last functions are from the formula (70) and the previous ones are the vector components that describe the magnetic field tension. Hence, when the system (69) is diagonalized, we can use the preceding results that were obtained for (47). Namely, in the second expression of (49″) for g_{j1} the values $f_j \quad (j=1,2),\ f_3$ change correspondingly to $\varphi_j \quad (j=1,2), \varphi_3$ from (70):

$$g_{j1} = A_{33}\varphi_j - A_{j3}\varphi_3 \quad (j=1,2).$$

After operators' and functions' substitutions from (48), (70) the last equality comes to

$$\begin{aligned} g_{11} &= (\hat{\partial}_0^* + \partial_3^2 - \Delta)\,(-\partial_3 j_2^{\text{ст}} + \partial_2 j_3^{\text{ст}}) - \partial_1\partial_3(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}), \\ g_{21} &= (\hat{\partial}_0^* + \partial_3^2 - \Delta)\,(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}) - \partial_2\partial_3(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}). \end{aligned} \tag{71}$$

Therefore, the three unknown scalar equations for $\{H_i\}_{i=1}^3$ have the left operator parts that are exactly identical to the appropriate parts of those scalar equations which are already obtained. Namely, the left parts of (49), (55) coincide with (51) and the left parts of (59) correspondingly

$$(\hat{\partial}_0^* - \Delta)^2\, \hat{\partial}_0^*\, (\hat{\partial}_0^* - \Delta + \partial_3^2)\, H_1 = g_{12}, \tag{72}$$

$$g_{12} = B_{22}^{(1)} g_{11} - B_{12}^{(1)} g_{21} \text{ (look (49′))}, \tag{72′}$$

where: g_{11}, g_{21} are from (71) and $B_{22}^{(1)}, B_{12}^{(1)}$ – from (50).

$$(\hat{\partial}_0^* - \Delta)^3\, (\hat{\partial}_0^* - \partial_1^2)\, \hat{\partial}_0^*\, (\hat{\partial}_0^* - \Delta + \partial_3^2)\, H_2 = h_1, \tag{73}$$

$$h_1 = B_{11}^{(2)} g_{21} - B_{21}^{(2)} h_0, \quad h_0 = g_{12} \text{ (look (57))}, \tag{73′}$$

where: functions g_{21}, g_{12} and operators $B_{11}^{(2)}, B_{21}^{(1)}$ are from the appropriate formulae (71), (72′) and (51), (50).

$$(\hat{\partial}_0^* - \Delta)^3\, \hat{\partial}_0^*\, (\hat{\partial}_0^* - \Delta + \partial_3^2)^2\, (\hat{\partial}_0^* - \partial_1^2) H_3 = h_2, \tag{74}$$

$$h_2 = B_{22}^{(1)}\left(B_{11}^{(2)} g_{30} - B_{31}^{(0)} h_0\right) - B_{32}^{(0)} h_1 \text{ (look (57))}, \tag{74′}$$

where: $B_{22}^{(1)}, B_{11}^{(2)}$ are from (50), (51); h_1 and h_0 are from (73′); $B_{3i}^{(0)} = A_{3i} \quad (i=\overline{1,3})$ and $g_{30} = f_3 = \varphi_3$ are defined in (58) and (70).

Now we can write the explicit expressions for the right parts of the desired scalar equations (72), (73), (74). These known functions are represented by the formulae (72′), (73′), (74′):

$$\begin{aligned} g_{12} &= (\hat{\partial}_0^* - \Delta)\,(\hat{\partial}_0^* - \partial_1^2)\left((\hat{\partial}_0^* + \partial_3^2 - \Delta)\,(-\partial_3 j_2^{\text{ст}} + \partial_2 j_3^{\text{ст}}) - \partial_1\partial_3(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}})\right) - \\ &- \partial_1\partial_2(\hat{\partial}_0^* - \Delta)\left((\hat{\partial}_0^* + \partial_3^2 - \Delta)\,(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}) - \partial_2\partial_3(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}})\right), \end{aligned} \tag{72″}$$

$$\begin{aligned} h_1 &= (\hat{\partial}_0^* - \Delta)^2\, \hat{\partial}_0^* (\hat{\partial}_0^* - \Delta + \partial_3^2)\left((\hat{\partial}_0^* + \partial_3^2 - \Delta)\,(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}) - \right. \\ &\left. - \partial_2\partial_3(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}})\right) - \partial_1\partial_2(\hat{\partial}_0^* - \Delta) g_{12}, \end{aligned} \tag{73″}$$

$$h_2 = (\hat{\partial}_0^* - \Delta)\,(\hat{\partial}_0^* - \partial_1^2)\left((\hat{\partial}_0^* - \Delta)^2\, \hat{\partial}_0^* (\hat{\partial}_0^* - \Delta + \partial_3^2)\, \varphi_3 - \partial_3\partial_1 g_{12}\right) - \partial_3\partial_2 h_1. \tag{74″}$$

Further, we transform the above written expressions (72″) – (74″) and substitute them consistently for one into another in the "chain" order (72″) → (73″) → (74″). Afterwards, we again transform (73″) and (74″) to the more compact forms, i.e.: at first we write

$$\begin{aligned} g_{12} &= (\hat{\partial}_0^* - \Delta)\left((\hat{\partial}_0^* + \partial_3^2 - \Delta)\left((\hat{\partial}_0^* - \partial_1^2)\,(-\partial_3 j_2^{\text{ст}} + \partial_2 j_3^{\text{ст}}) - \partial_1\partial_2(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}})\right) - \right. \\ &\left. - \partial_1\partial_3\left((\hat{\partial}_0^* - \partial_1^2)\,(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}) + \partial_2^2(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}})\right)\right). \end{aligned}$$

After application to the last formula the following expression

$$-(\hat{\partial}_0^* + \partial_3^2 - \Delta)\left(\partial_3(\hat{\partial}_0^* - \partial_1^2) j_2^{\text{ст}} + \partial_1\partial_2\partial_3 j_1^{\text{ст}} - \partial_2\, \hat{\partial}_0^*\, j_3^{\text{ст}}\right) - \partial_1\partial_3\left(-\partial_2(\hat{\partial}_0^* - \partial_1^2 - \partial_2^2)\, j_1^{\text{ст}} + \partial_1(\hat{\partial}_0^* - \partial_1^2 - \partial_2^2) j_2^{\text{ст}}\right)$$

and according to (44) g_{12} turns into

$$g_{12} = -(\hat{\partial}_0^* - \Delta)\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,(\partial_3\,\hat{\partial}_0^*\, j_2^{\text{ст}} - \partial_2\,\hat{\partial}_0^*\, j_3^{\text{ст}}) = -(\hat{\partial}_0^* - \Delta)\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*(\partial_3 j_2^{\text{ст}} - \partial_2 j_3^{\text{ст}}). \tag{75}$$

Then we insert (75) into the right part of (73″) and after obvious operator transition we obtain the explicit formula for h_1

$$h_1 = (\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*\big((\hat{\partial}_0^* + \partial_3^2 - \Delta)\,(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}) - \partial_2\partial_3(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}) + \partial_1\partial_2(\partial_3 j_2^{\text{ст}} + \partial_2 j_3^{\text{ст}})\big)$$

that according to (44) is equivalent to the following

$$\begin{aligned} h_1 &= (\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*\big(\partial_3(\hat{\partial}_0^* - \partial_1^2)\, j_1^{\text{ст}} - \partial_1(\hat{\partial}_0^* - \partial_1^2)\, j_3^{\text{ст}}\big) = \\ &= (\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \partial_1^2)\,(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}). \end{aligned} \tag{76}$$

At last, we substitute (75), (76) and φ_3 from the formula (70) for (74‴):

$$\begin{aligned} h_2 &= (\hat{\partial}_0^* - \Delta)\,(\hat{\partial}_0^* - \partial_1^2)\,\big((\hat{\partial}_0^* - \Delta)^2\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\big(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}\big) + \\ &+ \partial_3\partial_1(\hat{\partial}_0^* - \Delta)\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*(\partial_3 j_2^{\text{ст}} - \partial_2 j_3^{\text{ст}})\big) - \\ &- \partial_3\partial_2(\hat{\partial}_0^* - \Delta)^2\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \partial_1^2)\,(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}). \end{aligned}$$

After evident calculations the last expression comes to

$$\begin{aligned} h_2 &= (\hat{\partial}_0^* - \Delta)^2(\hat{\partial}_0^* - \partial_1^2)\,(\hat{\partial}_0^* + \partial_3^2 - \Delta)\,\hat{\partial}_0^*\big((\hat{\partial}_0^* - \Delta)\,\big(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}\big) + \\ &+ \partial_3\partial_1(\partial_3 j_2^{\text{ст}} - \partial_2 j_3^{\text{ст}}) - \partial_3\partial_2(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}})\big) \end{aligned}$$

$$\Updownarrow$$

$$\begin{aligned} h_2 &= (\hat{\partial}_0^* - \Delta)^2(\hat{\partial}_0^* - \partial_1^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\big(-\partial_2(\hat{\partial}_0^* - \Delta + \partial_3^2)\, j_1^{\text{ст}} + \partial_1\big(\partial_3^2 + \hat{\partial}_0^* - \Delta\big)\, j_2^{\text{ст}}\big) = \\ &= (\hat{\partial}_0^* - \Delta)^2(\hat{\partial}_0^* - \partial_1^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2)^2\,(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}). \end{aligned} \tag{77}$$

Then we insert (75) – (77) to the right parts of the corresponding equations (72) – (74)

$$\begin{aligned} &(\hat{\partial}_0^* - \Delta)^2\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2)\,H_1 = -(\hat{\partial}_0^* - \Delta)\,(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*(\partial_3 j_2^{\text{ст}} - \partial_2 j_3^{\text{ст}}), \\ &(\hat{\partial}_0^* - \Delta)^3(\hat{\partial}_0^* - \partial_1^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2)\,H_2 = (\hat{\partial}_0^* - \Delta)^2(\hat{\partial}_0^* - \Delta + \partial_3^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \partial_1^2)\,(\partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}), \\ &(\hat{\partial}_0^* - \Delta)^3\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2)\,(\hat{\partial}_0^* - \partial_1^2)\,H_3 = (\hat{\partial}_0^* - \Delta)^2(\hat{\partial}_0^* - \partial_1^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2)\,(-\partial_2 j_1^{\text{ст}} + \partial_1 j_2^{\text{ст}}) \end{aligned}$$

and neglect the corresponding “common operator multipliers”

$$\begin{aligned} &(\hat{\partial}_0^* - \Delta)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2), \\ &(\hat{\partial}_0^* - \Delta)^2(\hat{\partial}_0^* - \partial_1^2)\,\hat{\partial}_0^*(\hat{\partial}_0^* - \Delta + \partial_3^2), \\ &(\hat{\partial}_0^* - \Delta)^2\,\hat{\partial}_0^*(\hat{\partial}_0^* - \partial_1^2)\,(\hat{\partial}_0^* + \partial_3^2 - \Delta). \end{aligned} \tag{78}$$

As the result we obtain the sought for scalar equations

$$\begin{aligned} &(\hat{\partial}_0^* - \Delta)\,H_1 = \partial_2 j_3^{\text{ст}} - \partial_3 j_2^{\text{ст}}, \\ &(\hat{\partial}_0^* - \Delta)\,H_2 = \partial_3 j_1^{\text{ст}} - \partial_1 j_3^{\text{ст}}, \\ &(\hat{\partial}_0^* - \Delta)\,H_3 = \partial_1 j_2^{\text{ст}} - \partial_2 j_1^{\text{ст}} \end{aligned} \tag{79}$$

with respect to the components $\{H_i\}_{i=1}^3$ of the unknown vector-function $\vec{H}$ that describes the magnetic field tension.

The above written equations (79) jointly with (67) complete the present part 4 and confirm the special results of paper [10].

In the conclusion of given paper it should be noted that the proposed diagonalization procedure does not need any concrete initial and boundary conditions which become necessary only when the obtained scalar equations have to be solved, i.e. when the diagonalization algorithm is finished completely. Also we have to remind that the general approach from the parts 2, 3 does not require any additional conditions in the original system (4), except the operator commutativity in pairs (5). Besides, as we mentioned at the beginning of part 4, the proposed method may be applied to the matrix operators of the arbitrary block structure. In this case we construct the diagonalized block matrix at first, then apply the diagonalization procedure from block-to-block consistently and from the external operator elements to the inner ones, until we obtain the unknown scalar equations with the components of the original vector-function $\vec{F}$.

In other words, the considered method does not depend neither on the operator matrix structure nor on the initial and boundary conditions of the studied problem (4).

Our last remark concerns the investigated example from the part 4. It is easy to notice that the orders of scalar equations at the final diagonalization stage were essentially reduced when the corresponding "common operator multipliers" were neglected. Namely, — (53), (61), (65), (79) in the coordinate case (for the block stage it was (42)). After described operation we came to the equation that was equivalent to the preceding one, though the above proposed simplification way seemed formal from the first sight. In reality, neglect of the "common operator multiplier" completely coordinates with the given diagonalization algorithm. Really, when we applied to the both parts of the corresponding equation one and the same operator that was raised by the initial operators from (4) and then summed the transformed equations, we obtained the new system which was equivalent to the previous one. This system had one of the initial equations without recently applied operator and also the final equation, as the result of mentioned transformed equations' addition. In other words, the proposed procedure in the given paper represents the operator analogue of the algebraic systems' solution.

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