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A SIMPLE METHOD FOR A PRECISE SOLUTION OF THE DIGITAL OPTIMAL CONTROLLERS DESIGN PROBLEM FOR SISO OBJECTS WITH DELAY

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ПРОСТИЙ МЕТОД ОТРИМАННЯ ТОЧНОГО РОЗВ'ЯЗКУ ЗАДАЧІ СИНТЕЗУ ЦИФРОВИХ ОПТИМАЛЬНИХ РЕГУЛЯТОРІВ ДЛЯ ОДНОВИМІРНИХ ОБ'ЄКТІВ З ЗАПІЗНЕННЯМ

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ПРОСТОЙ МЕТОД ПОЛУЧЕНИЯ ТОЧНОГО РЕШЕНИЯ ЗАДАЧИ СИНТЕЗА ЦИФРОВЫХ ОПТИМАЛЬНЫХ РЕГУЛЯТОРОВ ДЛЯ ОДНОМЕРНЫХ ОБЪЕКТОВ С ЗАПАЗДЫВАНИЕМ

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Abstract. Models of aperiodic and integrator links with delay are traditional representations of controlled objects of various processes. In this case, the PID-family algorithms are traditionally used as control algorithms for which standard technical means have been developed. At the same time, the appearance of powerful miniature controllers with unified interfaces predicates the control quality improvement in the base of more advanced digital control systems usage. Such systems can be designed using the developed approach for the synthesis of optimal digital controllers with observers. Such an approach, in the traditional formulation, is impossible without the usage of specialized software which often substantially overload computer resources. At the same time, it is desirable to have a simple tuning method for customizing the controller at the control object, as well as for use in adaptive control systems. Obtaining such rules is described in this article. The theoretical result that follows from the obtained rules is that the parameters of the optimal state regulator do not depend on the delay and are determined solely by the object time constant and the selected sample period.

Key words: system, delay, SISO, controller design, optimal controller, dead-beat observer, robustness, computer network.

Анотація. Моделі аперіодичних та інтегральних ланок із запізненням є традиційним представленням керованих об'єктів різних промислових процесів та систем телекомунікації і зв'язку. При цьому алгоритми ПІД-типу традиційно використовуються як алгоритми керування, для яких розроблені стандартні технічні засоби. У той самий час, поява потужних мініатюрних контролерів з уніфікованими інтерфейсами зумовлює можливість поліпшення якості управління на базі більш сучасних цифрових систем керування. Такі системи можуть бути розроблені з використанням розробленого підходу для синтезу оптимальних цифрових регуляторів зі спостерігачами. Використання такого підходу в традиційній постановці неможливе без використання спеціалізованих програм, які часто істотно завантажують комп'ютерні ресурси, але бажано мати простий метод для налаштування контролера на об'єкті технологічного керування, а також для використання в адаптивних системах керування. Отримання таких співвідношень описано в цій статті. Теоретичний результат, що випливає з отриманих співвідношень, полягає в тому, що параметри оптимального регулятора стану не залежать від запізнення і визначаються виключно постійною часу об'єкта і вибраним періодом дискретизації.

Ключові слова: система, запізнення, одновимірна, розрахунок регулятора оптимальний регулятор, аперіодичний спостерігач, робастність, комп'ютерна мережа.

Аннотация. Модели апериодических и интегральных звеньев с запаздыванием являются традиционным представлением управляемых объектов для промышленных процессов и систем телекоммуникации и связи. Для таких объектов в качестве алгоритмов управления традиционно используются регуляторы ПИД-семейства, для которых разработаны стандартные технические средства. В то же время, появление мощных миниатюрных контроллеров с унифицированными интерфейсами делает возможным улучшение качества управления на основе использования современных цифровых систем управления. Такие системы могут быть спроектированы с использованием разработанного подхода для синтеза оптимальных цифровых регуляторов с наблюдателями. Но такой подход в традиционной формулировке невозможен без использования специализированных программ, часто существенно загружающих компьютерные ресурсы. Было бы желательно иметь простой метод для настройки регулятора на объекте управления, а также для использования в адаптивных системах управления. Такие соотношения впервые описаны в статье. Теоретический результат, который следует из полученных соотношений, состоит в том, что параметры оптимального регулятора состояния не зависят от запаздывания и определяются исключительно постоянной времени объекта и выбранным периодом дискретности.

Ключевые слова: система, запаздывание, одномерная, проектирование регулятора, оптимальный регулятор, апериодический наблюдатель, робастность, компьютерная сеть.

General statement of the problem and its connection with important scientific or practical aims. In modern control theory evolution, many methods of ISE (Integral Square Error) optimal controller design have been developed [1]. Such controllers, in comparison with traditional PID-type controllers [2-5], have better control quality [6]. Theoretical and software tools for solving problems with such controllers have been developed [7]. But, for simple plants, such as widespread in industry and infocommunications control plants that can be described with the FOPDT (First Order Plus Dead Time) model, controller design algorithms are rather complex. So, the actual problem is the development of simplified control design algorithms for such plants that would provide optimal controllers that are widespread in industry.

Analysis of the latest research and publications, singling out previously unsolved parts of the general problem, which are the focal point of the article. Wide usage of computer networks demands creating modern digital controllers for control systems of these network loads. The problem of implementation of such control systems is described in monographs [8, 9]. The design problem of control systems for web-servers load is described in the article [10], for database servers load is described in the article [11], for cloud services load in the articles [12-14]. Usually PID control algorithms are used [8-14], but in most cases theese control systems are not optimal.

Statement of the aim of the article. The aim of the article is to develop alternative to the PID algorithm of digital optimal control for SISO systems with delay. The proposed algorithm allows the achievement of better quality of control and has a simple design procedure.

Laying out of the main research material. Let the mathematical model of the control object in continuous time be described by an aperiodic link of the first order with delay [15] in the form

$$W(s)=\frac{y(s)}{u(s)}=\frac{k}{T\cdot s+1}e^{-s\cdot\tau}, \qquad (1)$$

where k – gain; T – time constant; □ – delay; W – transfer function; $y(s)$ – object output; $u(s)$ – object input; s – Laplace variable.

Transforming the model to the digital form we obtain

$$y_{i+1}=a\cdot y_i+b\cdot u_{i-m}, \qquad (2)$$

where $a=e^{-\Delta t/T}$; $b=k\cdot(1-a)$; Δt – time step; $m=[\boldsymbol{\tau}/\Delta t]$; $y_i\overset{\Delta}{=}y(i\cdot\Delta t)$; $i=0,1,\ldots$.

Then the state-space model in standard form can be written as

$$x_{i+1}=A\cdot x_i+B\cdot u_i; \qquad y_i=C\cdot x_i, \qquad (3)$$

where

$$A=\begin{pmatrix} a & 0 & \ldots & 0 & 0 \\ 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & \ddots & \ldots & 0 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & 0 & 1 & 0 \end{pmatrix},\ B=\begin{pmatrix} b \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix},\ A\in R^{n,n},\ B\in R^{n,1},\ C\in R^{1,n},\ n=m+1.$$

$$C=\begin{pmatrix} 0 & 0 & \ldots & 0 & 1 \end{pmatrix}$$

It's known that an optimal controller can be written in the form [16]

$$\begin{gathered} x_{i+1}=(A-B\cdot K-L\cdot C)\cdot x_i+L\cdot(r-y_i), \\ u_i=-K\cdot x_i \end{gathered} \qquad (4)$$

where $K=(k_1 \ \ldots \ k_n)$ is a state space controller gain matrix; $L=(l_1 \ \ldots \ l_n)^T$ is a state observer matrix; r is a reference.

Let the performance index has the form

$$J=\sum_{i=0}^{\infty}x_i^{\,T}\cdot Q\cdot x_i+u_i^{\,T}\cdot R\cdot u_i\Rightarrow \min, \qquad (5)$$

where Q and R are weight positive semidefinite matrices. Then, the controller gain matrix is calculated by Riccati matrix equation

$$A^T \cdot P \cdot A - P - A^T \cdot P \cdot B \cdot \left(B^T \cdot P \cdot B + R\right)^{-1} \cdot B^T \cdot P \cdot A + Q = 0, \qquad (6)$$
$$K = \left(B^T \cdot P \cdot B + R\right)^{-1} \cdot B^T \cdot P \cdot A.$$

Simplifying we can write symbolically

$$[K,P] = lqr(A,B,Q,R). \qquad (7)$$

To simplify the solution we can write the weight matrix Q in diagonal form and R as a positive number

$$Q = \begin{pmatrix} q_1 & 0 & \dots & 0 \\ 0 & q_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q_n \end{pmatrix}, \quad q_i > 0, \quad R = \rho > 0. \qquad (8)$$

Then, using the solution (6, 7) for the object (3) with performance index (5) matrices (8) we can obtain the main result of the article, namely the state controller (4) matrix K has the form

$$\begin{aligned} &K = [h,0,\dots,0], \\ &[h,\rho] = lqr(a,b,tr(Q),\rho), \qquad tr(Q) = \sum_{j=1}^{n} q_j. \end{aligned} \qquad (9)$$

As we see, we obtain the simple scalar solution for complex matrix problem. Moreover, we see that the controller gain value h is not dependent on the delay value.

To reconstruct the whole matrix P, we can write

$$P = \begin{pmatrix} p_1 & 0 & \dots & 0 \\ 0 & p_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & p_n \end{pmatrix}, \qquad \begin{gathered} p_n = q_n, \\ p_{n-j} = p_{n-j+1} + q_{n-j}, \\ p_1 = p, \\ j = 1,\dots,n-2 \end{gathered} \qquad (10)$$

To prove the solution we can use direct substitution.

To obtain the complete control problem solution we should design the observer. To design it we use the approach for dead-beat controllers design and choose all eigenvalues of matrix $A - L \cdot C$ equal to zero. Then we can obtain matrix L in the form

$$L = \left[l_1,\dots,l_n\right], \quad l_j = a^{n-j+1}, \quad j = 1,\dots,n. \qquad (11)$$

To prove the solution we can use direct substitution.

We can use our results which are obtained for standard aperiodic link of the first order with a delay model (1) for standard integrator link of the first order with a delay model of the form

$$W(s) = \frac{y(s)}{u(s)} = \frac{k}{T \cdot s} e^{-s \cdot \tau}. \qquad (12)$$

Then we must use values $a=1$, $b=k\cdot\Delta t/T$ for the model in the form (3).

For example, we use the model

$$W(s)=\frac{3}{120\cdot s+1}e^{-s\cdot 80}.$$

Then, if we choose $\Delta t=10$ sec we obtain $n=9$. The weight matrices we choose in the form $q_j=1,\ j=1,\dots,8,\ \ q_9=10^7;\ R=1$.

Having done the calculations we obtain
K = [3.8356 0 0 0 0 0 0 0 0],
L^T = [0,4724 0,5134 0,5580 0,6065 0,6592 0,7165 0,7788 0,8465 0,9200].

We want to compare our controller functioning with standard P and PID [17, 18] controllers and to investigate the control system robustness. To compare controllers we tune these with standard Simulink tuner which is included in the Simulink controller blocks. The gain for P controller is K = 0,61336. The coefficients for ideal structure PID controller are P = 0,3057, I = 0,00945, D = 0. To investigate the robustness we change the object model coefficients in diapasons $T\pm 0.3\cdot T$; $\tau\pm 0.3\cdot\tau$; $k\pm 0.3\cdot k$.

The simulation model structure for investigation obtained controller properties is shown in the Fig. 1.

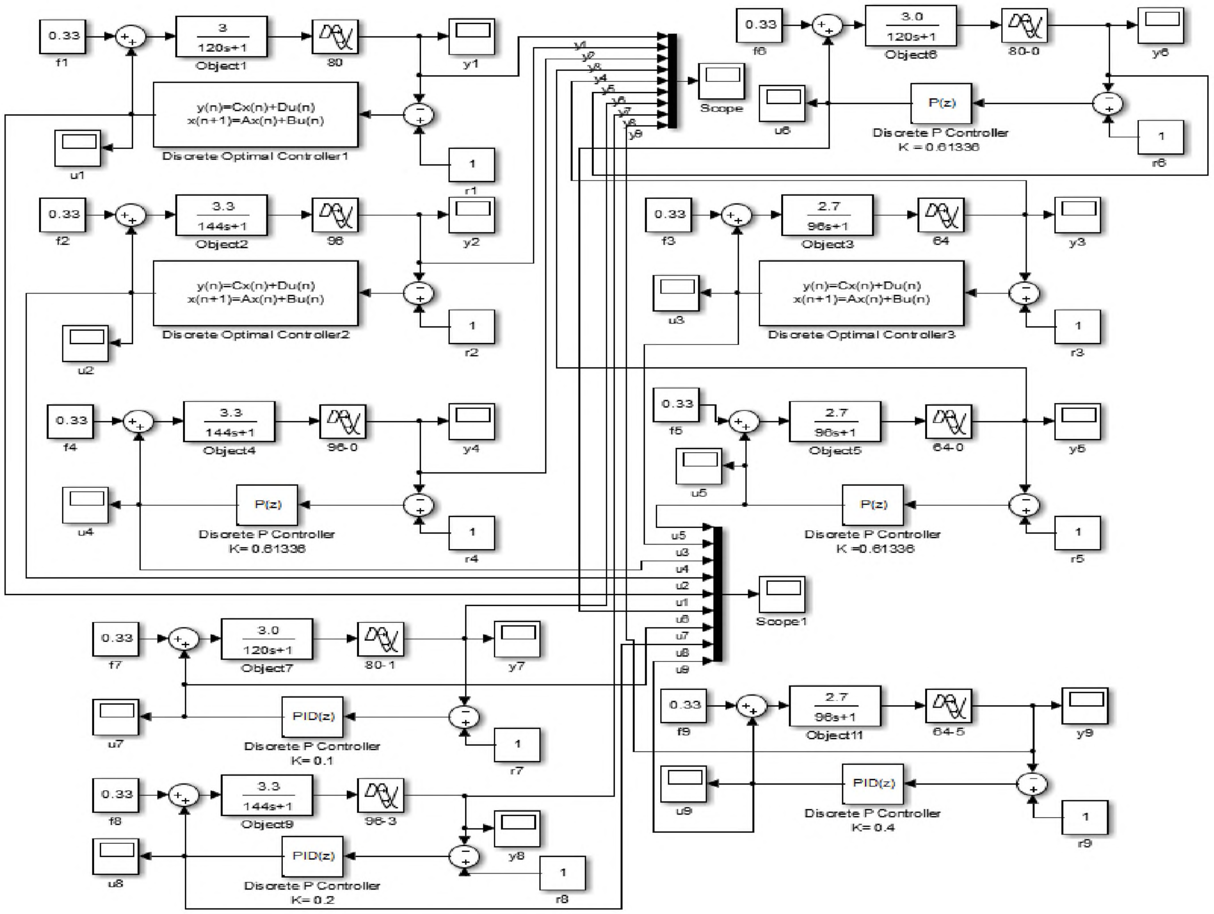

Figure 1 – The Simulink model structure for investigation obtained controller properties

The step responses of the investigated models (where $y_1 - y_9$ – outputs, $u_1 - u_9$ – controls) are shown on the Fig. 2 and 3. Having analyzed these responses we can come to the conclusion that the developed controller has the best properties and performance (lines y_1, y_2, y_3) according to the settling time and overshoot quality criteria.

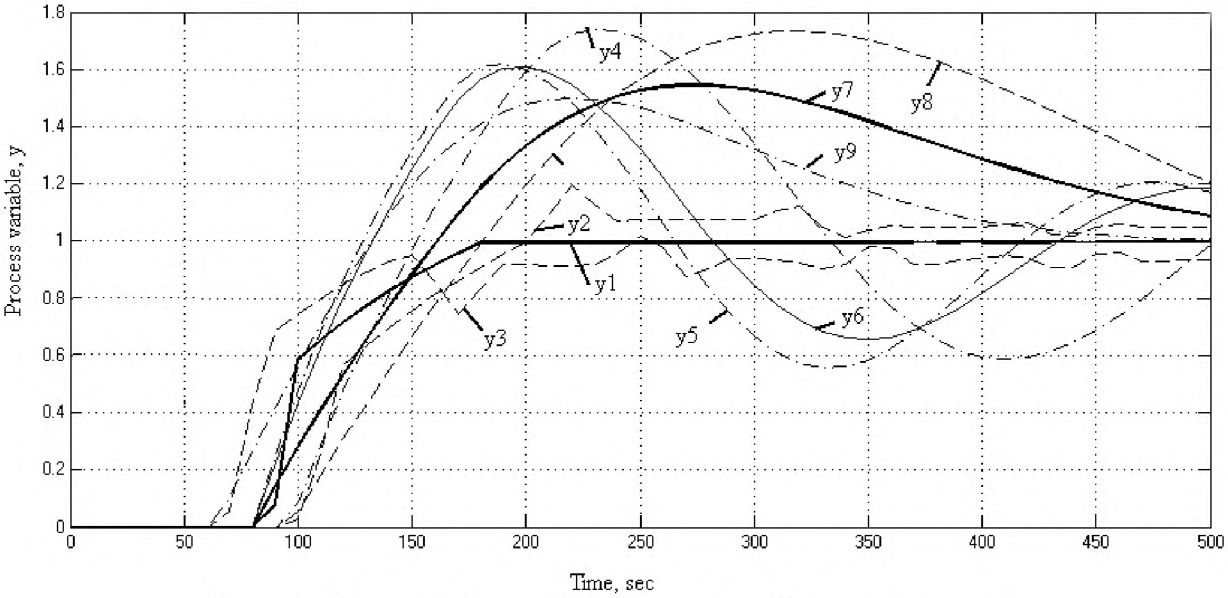

Figure 2 – The model outputs step responses for reference r = 1

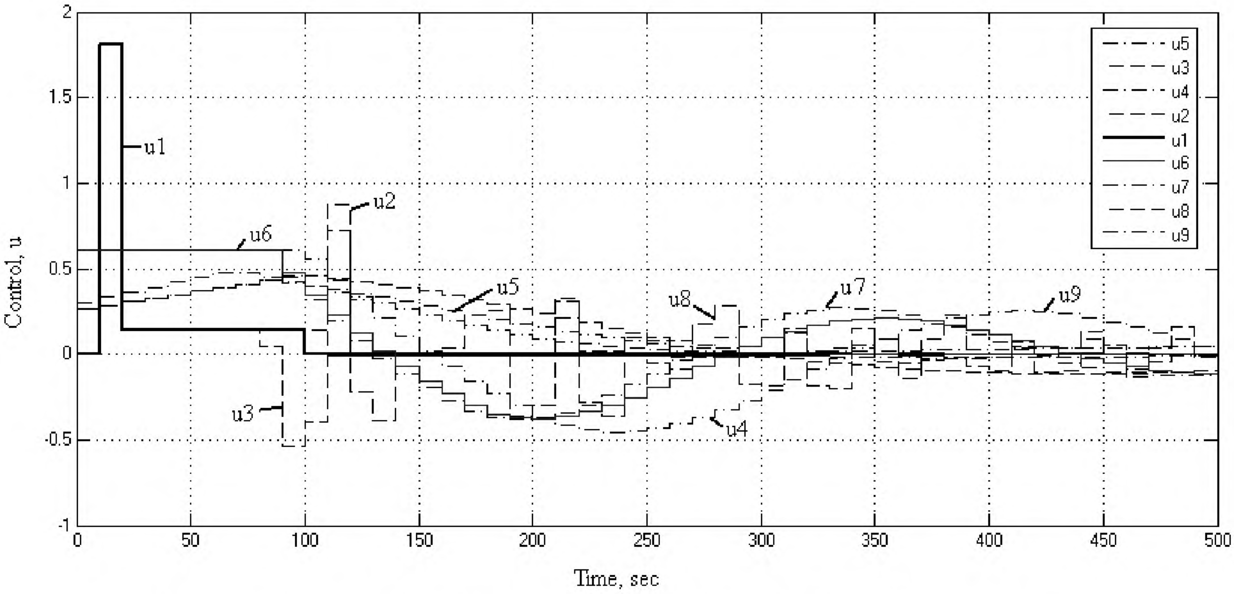

Figure 3 – The model digital controls step responses for reference r = 1

We can point out that if we choose Δt = 1 sec we obtain the object order n = 81 and the design time for standard optimal approach (6) and our approach is related as 10^4 times [19, 20].

Conclusions of the research and perspectives of further studies in this area. In conclusion we can say that a new approach for optimal digital control systems synthesis is suggested. The approach lets us calculate controller parameters (controller and observer matrices) substantially faster. The calculation of the matrices can be executed without a computer. Also, it is shown that the parameters of the optimal state controller is not dependent on delay but depends on the object time constant and sample time only. The delay determines the dimension of controller and observer matrices. Further studies should be devoted to research of the robustness of the developed control systems.

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