

## NOVEL 2-12QAM MODULATION FORMAT

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## НОВИЙ ФОРМАТ МОДУЛЯЦІЇ 2-12QAM

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## НОВЫЙ ФОРМАТ МОДУЛЯЦИИ 2-12QAM

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**Abstract.** The 16 Quadrature Amplitude Modulation (16QAM) is the preferred modulation format for many communication and broadcasting standards. However, performance of the overall “practical” communication/ broadcasting systems employing 16QAM is far from the Shannon limit as there is a requirement for power back-up to combat non-linear distortions generated by the output of high power amplifiers. In this paper we propose moving from 16QAM to a 12QAM modulation without any change of the hardware and with only minor changes in the mapping algorithm, by eliminating the four constellation points with highest signal energy. However, to ensure that no data is lost and spectral efficiency is preserved, we propose pairing two 12QAM symbols into a 4-dimensional signal with the appropriate mapping, obtaining the desired 2-12QAM modulation. We propose the use of a rate 6/7 forward error correcting (FEC) code combined with the 2-12QAM, which maps 7 binary input digits into 2 twelve-ary symbols which select corresponding signals from the conventional 16QAM modulator. The proposed new modulation format has a spectral efficiency of 3 bits/symbol, the same spectral efficiency as the legacy 16QAM scheme when the 16QAM modulation format is coupled with a rate 3/4 FEC. The removal of the 16QAM highest energy symbols represents a peak energy gain of 2.55 dB and an average energy gain of 1,35 dB which is offset by a 0.58 dB increase in code rate, from 3/4 to 6/7. In addition, the proposed scheme has a clear advantage of employing existing 16QAM modulator/demodulator hardware and maintaining compatibility with the legacy standards, albeit with simple changes in mapping which can be achieved via a software upgrade. We show that the proposed technique allows at least 2dB power gain without sacrificing spectral efficiency or additional hardware complexity.

**Key words:** Non-linearity mitigation, BCJR algorithm, Forward error correction.

**Анотація.** 16-квадратурна амплітудна модуляція (16QAM) є кращим форматом модуляції для багатьох стандартів зв'язку та мовлення. Тим не менш, продуктивність загальних «практичних» систем зв'язку/мовлення, що використовують 16QAM, далеко від межі Шеннона, оскільки існує вимога до резервного живлення для боротьби з нелінійними спотвореннями, що генеруються виходом підсилювачів високої потужності. У даній статті ми пропонуємо перейти від 16QAM до 12QAM модуляції без будь-якої зміни апаратного забезпечення і лише з незначними змінами в алгоритмі відображення, усунувши чотири точки сузір'я з найвищою енергією сигналу. Однак, щоб гарантувати, що ніякі дані не втрачаються і спектральна ефективність зберігається, ми пропонуємо поєднання двох

12QAM символів у 4-мірний сигнал з відповідним відображенням, отримуючи бажану 2-12QAM модуляцію. Ми пропонуємо використовувати код швидкості 6/7 для корекції помилок (FEC) в поєднанні з 2-12QAM, який відображає 7 двійкових вхідних цифр у 2 дванадцятирічних символах, які вибирають відповідні сигнали від звичайного 16QAM модулятора. Запропонований новий формат модуляції має спектральну ефективність 3 біта/символ, таку ж спектральну ефективність, що й успадкована схема 16QAM, коли формат модуляції 16QAM з'єднаний зі швидкістю 3/4 FEC. Видалення символів найвищої енергії 16QAM являє собою піковий приріст енергії 2,55 дБ і середній приріст енергії 1,35 дБ, який компенсується збільшенням кодової швидкості на 0,58 дБ, з 3/4 до 6/7. Крім того, запропонована схема має чітку перевагу використання існуючого забезпечення модулятора/демодулятора 16QAM і підтримки сумісності з традиційними стандартами, хоча і з простими змінами у відображенні, які можуть бути досягнуті за допомогою оновлення програмного забезпечення. Ми показуємо, що запропонована методика дозволяє отримати принаймні не менше 2 дБ посилення потужності без шкоди для спектральної ефективності або додаткової апаратної складності.

**Ключові слова:** зменшення лінійності, алгоритм BCJR, пряме виправлення помилок.

**Анотація.** 16-квадратурная амплитудная модуляция (16QAM) является предпочтительным форматом модуляции для многих стандартов связи и вещания. Однако производительность общих «практических» систем связи/вещания, использующих 16QAM, далека от предела Шеннона, поскольку существует потребность в резервировании мощности для борьбы с нелинейными искажениями, генерируемыми выходом усилителей большой мощности. В этой статье мы предлагаем перейти от 16QAM к модуляции 12QAM без каких-либо изменений аппаратного обеспечения и с незначительными изменениями в алгоритме отображения, исключив четыре точки созвездия с самой высокой энергией сигнала. Однако, чтобы гарантировать, что никакие данные не будут потеряны и сохранена спектральная эффективность, мы предлагаем объединить два символа 12QAM в 4-мерный сигнал с соответствующим отображением, получая желаемую модуляцию 2-12QAM. Мы предлагаем использовать код прямого исправления ошибок (FEC) со скоростью 6/7 в сочетании с 2-12QAM, который отображает 7 двоичных входных цифр в 2 двенадцатиричных символах, которые выбирают соответствующие сигналы из традиционного модулятора 16QAM. Предложенный новый формат модуляции имеет спектральную эффективность 3 бита/символа, такую же спектральную эффективность, что и унаследованная схема 16QAM, когда формат модуляции 16QAM связан со скоростью 3/4 FEC. Удаление символов 16QAM с наивысшей энергией представляет пиковый прирост энергии 2,55 дБ и средний прирост энергии 1,35 дБ, что компенсируется увеличением кодовой скорости на 0,58 дБ, с 3/4 до 6/7. Кроме того, предложенная схема имеет явное преимущество использования существующего аппаратного обеспечения модулятора/демодулятора 16QAM и поддержки совместимости с существующими стандартами, хотя и с простыми изменениями в отображении, которые могут быть достигнуты посредством обновления программного обеспечения. Мы показываем, что предлагаемый метод позволяет получить усиление мощности не менее 2 дБ без ущерба для спектральной эффективности или дополнительной аппаратной сложности.

**Ключевые слова:** уменьшение нелинейности, алгоритм BCJR, прямое исправление ошибок.

The 16 Quadrature Amplitude Modulation (16QAM) is a de-facto preferred modulation format for many communication and broadcasting standards, such as IEEE 802.16, 3GPP and DVB-S2 [1], just to name a few. 16QAM provides good performance and in some cases, for example, when combined with capacity approaching codes, has shown channel performance close to the Shannon limit [2]. However, performance of the overall “practical” communication/ broadcasting systems employing 16QAM is far from the Shannon limit as there is a requirement for power back-up to combat non-linear distortions generated by the output of high power amplifiers. In some systems, such as satellite communications and digital TV broadcasting, this requires up to 10 dB of power back-up, thus significantly reducing overall system efficiency [3]. Numerous techniques, such as non-linearity correction, dynamic pre-distortion, peak-to average ratio reduction, and others for combating this negative effect were introduced [4-12]. However, their implementation requires significant changes in the overall system and raises questions of compatibility with the legacy standards. In this Paper we propose a different approach, where the legacy 16QAM modulation is used to generate 4-dimensional 2-12QAM modulation by eliminating four 16QAM symbols with highest energy without any change of the hardware and with only minor changes in the mapping algorithm. We show that up to 2 dB energy gain can be achieved while maintaining compatibility with the legacy systems.

In a 16QAM system [13] signal amplitude and phase vary according to the digital information to be transmitted. The signals  $s_i(t), 1 \leq i \leq 16$ , are represented as

$$s_i(t) = A_x \cos(2\pi f_c t) + B_y \sin(2\pi f_c t), 0 \leq t \leq T,$$

where  $A_x$  and  $B_y$  are amplitudes allowed in the  $\cos(2\pi f_c t)$  and  $\sin(2\pi f_c t)$  axis, respectively,  $f_c$  is the carrier frequency,  $T$  is the duration of each symbol,  $A_x \in \{(2x - \sqrt{M} - 1)d\}_{x=1}^{\sqrt{M}}$ ,  $A_y \in \{(2y - \sqrt{M} - 1)d\}_{y=1}^{\sqrt{M}}$  and  $d$  is the distance between two adjacent signals. The signal constellation [1] of 16QAM is shown in Figure 1A) and consists of 16 2-dimensional vectors  $s_i, 1 \leq i \leq 16$ , denoted as

$$s_i = [A_x, B_y]^T, \quad i = 1, \dots, 16.$$

The legacy 16QAM modulation format combined with a rate 3/4 forward error-correcting (FEC) code is shown in Fig. 1B). Conventional mapping is employed to transfer four binary bits into one 16-ary symbol from the 16QAM constellation, followed by a power amplifier (PA). The 16QAM modulation format coupled with a rate 3/4 FEC code has a spectral efficiency of 3bits/symbol. While this diagram is widely used in various communication systems, it has one significant drawback. Due to non-linearity of the PA the overall system requires power back-up leading to a significant reduction in the overall system efficiency [1].

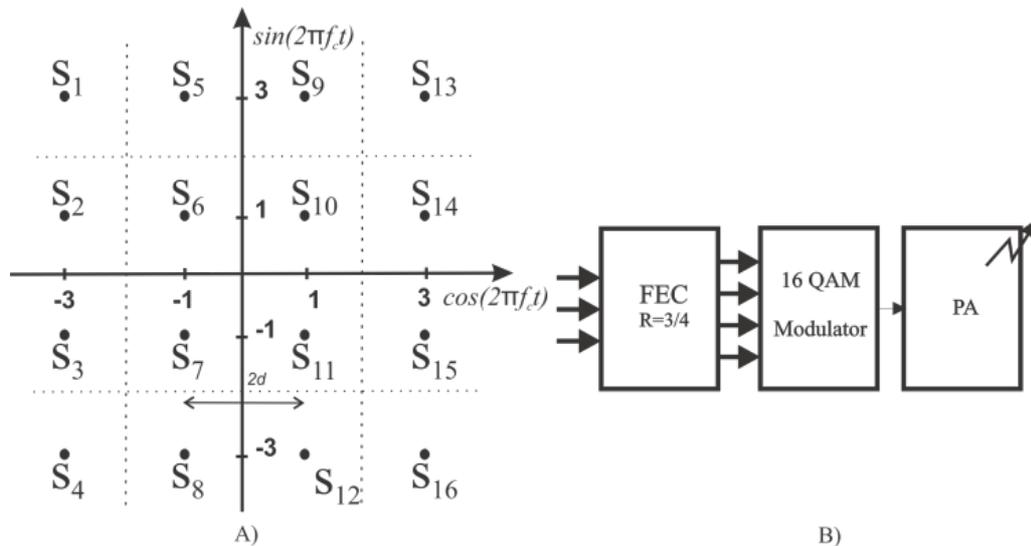


Figure 1 – a) 16QAM constellation and decision regions; б) Block diagram of 16QAM system with rate 3/4 FEC code

**The 2-12QAM.** To mitigate the drawback mentioned earlier, we propose to eliminate the four constellation points with highest signal energy (indicated in Figure 1 as signals  $S_1, S_4, S_{13}$  and  $S_{16}$ ), moving from 16QAM to a 12QAM modulation. However, to ensure that no data is lost and spectral efficiency is preserved, we propose pairing of two 12QAM symbols into a 4-dimensional signal with the appropriate mapping, as shown in Fig. 2. To achieve the desired result, we exploit an approach used for generating non-binary line codes with special spectral shaping characteristics [14], as illustrated in Fig. 3. In this diagram we use a rate 6/7 FEC code and a novel 7 binary to 2 twelve-ary (7B-2Tw) mapper, which maps 7 binary input digits into 2 twelve-ary symbols which select corresponding signals from the conventional 16QAM modulator (eliminating four signals with the highest energy). Since  $27 < 122$ , there is sufficient redundancy to complete this mapping.

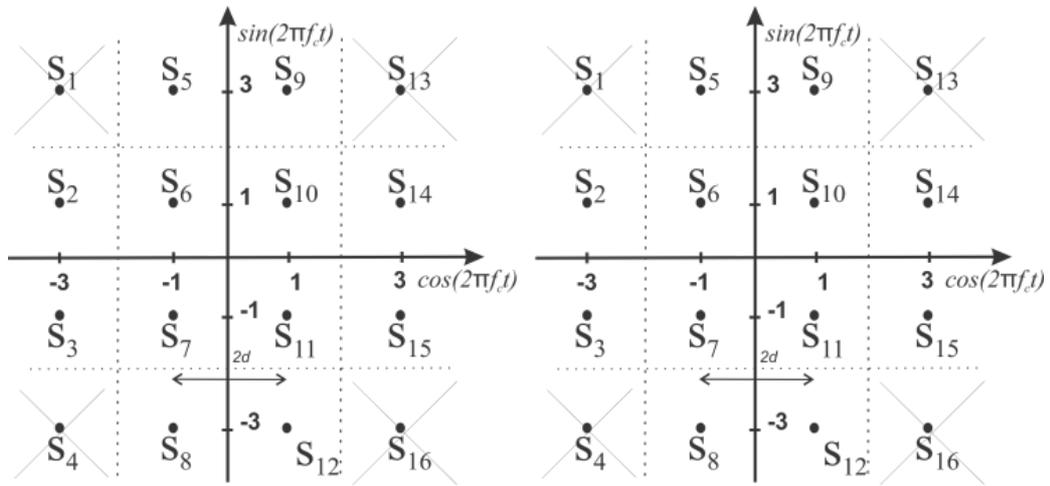


Figure 2 – Diagram of a 4-dimensional 2-12QAM and decision regions

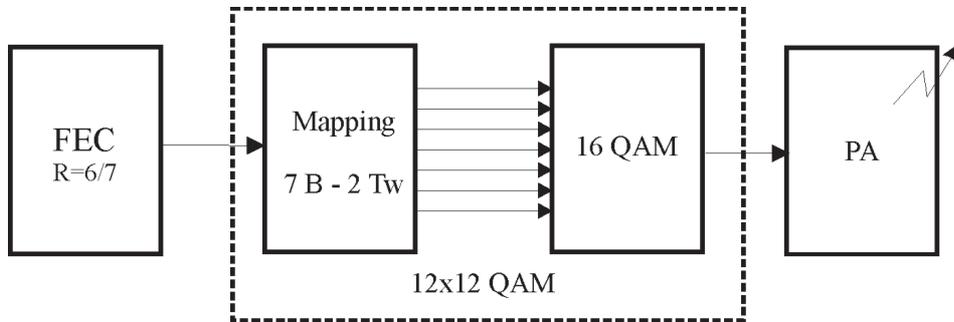


Figure 3 – Block diagram of a 4-dimensional 2-12QAM constellation with a rate 6/7 FEC code employing the legacy 16QAM scheme

As follows from Fig. 3, the proposed new modulation format has a spectral efficiency of 3 bits/symbol, i.e., the same spectral efficiency as the legacy 16QAM scheme. The removal of the 16QAM highest energy symbols represent a peak energy gain of 2,55 dB and an average energy gain of 1,35 dB which is offset by a 0,58 dB increase in code rate, from 3/4 to 6/7. In addition, the proposed scheme has a clear advantage of employing existing 16QAM modulator/demodulator hardware and maintaining compatibility with the legacy standards, albeit with simple changes in mapping which can be achieved via a software upgrade.

In the proposed 2-12QAM the signal  $s_{ij}(t)$ ,  $0 < t \leq 2T$ , is given by

$$s_{ij}(t) = \begin{cases} A_x \cos(2\pi f_c t) + B_y \sin(2\pi f_c t), & 0 < t \leq T \\ C_s \cos(2\pi f_c t) + D_z \sin(2\pi f_c t), & T < t \leq 2T, \end{cases}$$

where  $i \in \{2,3,5,6,7,8,9,10,11,12,14,15\}$ ,  $j \in \{2,3,5,6,7,8,9,10,11,12,14,15\}$  and  $A_x$  and  $B_y$  are amplitudes allowed in the  $\cos(2\pi f_c t)$  and  $\sin(2\pi f_c t)$  axis shown in Fig. 2, a) and  $C_s$  and  $D_z$  are amplitudes allowed in the  $\cos(2\pi f_c t)$  and  $\sin(2\pi f_c t)$  axis shown in Fig. 2, б), and  $f_c$  is the carrier frequency. The 4-dimensional structure is composed of two 2-dimensional diagrams illustrated in Figure 2, where the corner regions were removed from the usual 16QAM diagram in order to reduce peak power. The signal constellation consists of 128 ( $2^7$ ) 4-dimensional vectors, chosen from among 144 ( $12^2$ ) 4-dimensional possible vectors, where each constellation symbol  $s_{i,j}$  represents 7 information bits that we denote as  $s_{i,j} \in \{[A_x, B_y]^T, [C_s, D_z]^T\}$ , where

$$\begin{aligned} i &\in \{2,3,5,6,7,8,9,10,11,12,14,15\}; \\ j &\in \{2,3,5,6,7,8,9,10,11,12,14,15\}. \end{aligned}$$

The corresponding probability density function  $p_{\mathbf{r}/H_{i,j}}(\mathbf{r})$  is given by

$$p_{\mathbf{r}/H_{i,j}}(\mathbf{r}) = \left(\frac{1}{\pi N_0}\right)^2 \exp\left[-\frac{(r_1 - A_x)^2 + (r_2 - B_y)^2}{N_0}\right] \exp\left[-\frac{(r_3 - C_s)^2 + (r_4 - D_z)^2}{N_0}\right].$$

Supposing that  $H_{i,j}$  is the correct hypothesis, the received vector is given by

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} A_x + n_1 \\ B_y + n_2 \\ C_s + n_3 \\ D_z + n_4 \end{bmatrix},$$

where  $n_1, n_2, n_3$  and  $n_4$  are Gaussian noise samples, i.e. Gaussian random variables, with mean  $\mu = 0$  and variance  $\sigma^2 = N_0/2$ . In order to simplify notation we will write the complementary error function of Gaussian statistics  $Q(d/\sigma) \triangleq (1/\sqrt{2\pi}) \int_{d/\sigma}^{\infty} e^{-y^2/2} dy$  [2] to mean the function  $Q$ . Considering equiprobable signals, the probability of correct decision is given by  $P(C) = \frac{1}{128} \sum_{i,j} P(C/H_{i,j})$ , where the conditional probability  $P(C/H_{i,j})$ , can assume three distinct values. In fact, we have three groups, with the constellation symbols represented by the pairs  $(i, j)$ , having different energies:

1) If  $(i, j)$  is in the set with the highest energy, as for example the pair  $(i, j) = (2, 2)$ , then  $P(C/H_{i,j}) = P(-d < n_1 < d)P(n_2 > -d)P(n_4 > -d)$  or, equivalently,  $P(C/H_{i,j}) = (1 - 2Q)^2(1 - Q)^2$ .

2) If  $(i, j)$  is in the set with the medium energy, as for example the pair  $(i, j) = (2, 6)$ , then  $P(C/H_{i,j}) = P(-d < n_1 < d)P(n_2 > -d)P(-d < n_3 < d)P(-d < n_4 < d) = P(-d < n_1 < d)P(-d < n_2 < d)P(-d < n_3 < d)P(n_4 > -d)$  or, equivalently,  $P(C/H_{i,j}) = (1 - 2Q)^3(1 - Q)$ .

3) If  $(i, j)$  is in the set with the lowest energy, as for example the pair  $(i, j) = (6, 6)$ , then  $P(C/H_{i,j}) = P(-d < n_1 < d)P(-d < n_2 < d)P(-d < n_3 < d)P(-d < n_4 < d)$ , or equivalently,  $P(C/H_{i,j}) = (1 - 2Q)^4$ .

The probability of correct decision is given by

$$P(C) = \frac{1}{128} [48(1 - 2Q)^2(1 - Q)^2 + 64(1 - 2Q)^3(1 - Q) + 16(1 - 2Q)^4], \quad (1)$$

where the 48 pairs from the first term in (1) are selected from the set 1).

**The proposed system using convolutional encoder and BCJR decoding algorithm.** For simplicity, consider that in Figure 3 it is used a systematic convolutional encoder with rate  $R = \frac{1}{2}$  and  $M$  states. The information symbols sequences are given by  $\mathbf{u} = u_0^{\{N-1\}} = \{u_0, u_1, \dots, u_{\{N-1\}}\}$  and the code sequence associated is  $\mathbf{v} = v_0^{\{N-1\}} = \{v_0, v_1, \dots, v_{\{N-1\}}\}$ , where  $\mathbf{v}_t = (v_t^{(1)}, v_t^{(2)}) = (u_t, v_t^{(2)})$  is the output related with each information symbol. The code rate is changed by the use of a puncturer having puncturing matrix [15] given by

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (2)$$

The new code has rate  $R = \frac{6}{7}$  and the code sequence  $\mathbf{w}_t$  is given by

$$\begin{cases} \mathbf{w}_t = (u_t, v_t^{(2)}), & t = 6 \cdot b, b = \{0, 1, 2, \dots, \frac{N}{6} - 1\} \\ \mathbf{w}_t = (u_t), & \text{others, } t < N. \end{cases}$$

It means that

$$\begin{aligned} \{u_0 v_0^{(2)}, u_1, u_2, u_3, u_4, u_5\} &= \{\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4, \mathbf{w}_5\}; \\ \{u_6 v_6^{(2)}, u_7, u_8, u_9, u_{10}, u_{11}\} &= \{\mathbf{w}_6, \mathbf{w}_7, \mathbf{w}_8, \mathbf{w}_9, \mathbf{w}_{10}, \mathbf{w}_{11}\}; \\ &\vdots \\ \{u_{\{N-6\}} v_{N-6}^{(2)}, u_{N-5}, u_{N-4}, u_{N-3}, u_{N-2}, u_{N-1}\} &= \{\mathbf{w}_{N-6}, \mathbf{w}_{N-5}, \mathbf{w}_{N-4}, \mathbf{w}_{N-3}, \mathbf{w}_{N-2}, \mathbf{w}_{N-1}\}. \end{aligned}$$

#### A. The adapted BCJR algorithm

The code sequence is modulated by the 2-12QAM and is transmitted through additive white Gaussian noise channel (AWGN). The decoder using the Bahl, Cocke, Jelinek and Raviv (BCJR) decoding algorithm is adapted for this system and employed. For each six time slots the channel output is the received sequence given by  $\mathbf{R}_k$ , where

$$\mathbf{R}_K = \begin{bmatrix} r_{1K} \\ r_{2K} \\ r_{3K} \\ r_{4K} \end{bmatrix} = \begin{bmatrix} A_{xK} + n_{1K} \\ B_{yK} + n_{2K} \\ C_{sK} + n_{3K} \\ D_{zK} + n_{4K} \end{bmatrix},$$

$$\mathbf{R} = \mathbf{R}_0^{\left(\frac{N}{6}-1\right)} = \left(\mathbf{R}_0, \mathbf{R}_1, \dots, \mathbf{R}_{\frac{N}{6}-1}\right),$$

where  $n_{1K}, n_{2K}, n_{3K}$  and  $n_{4K}$  are Gaussian noise samples, with mean  $\mu = 0$  and variance  $\sigma^2 = N_0/2$ .

We employ the trellis associated with the convolutional encoder and compute the log-likelihood ratio

$$\Lambda(u_t) = \log \frac{P(u_t=1/R)}{P(u_t=0/R)},$$

where  $P(u_t=a/R)$ ,  $a = 0, 1$  is the *a posteriori* probability of the information symbol  $u_t$ . For  $0 \leq t \leq N - 1$ , where  $N$  is the length of the code sequence, the decision is done in the following way

$$\begin{cases} \Lambda(u_t) \geq 0: \hat{u}_t = 1; \\ \Lambda(u_t) < 0: \hat{u}_t = 0. \end{cases}$$

Let  $S_t$  denotes the trellis state at time slot  $t$  and  $p\{x\}$  the probability density function of  $x$ . It follows that

$$\Lambda(u_t) = \log \frac{\sum_m \sum_{m'} p\{u_t=1, S_t=m, S_{t-1}=m', \mathbf{R}\}}{\sum_m \sum_{m'} p\{u_t=0, S_t=m, S_{t-1}=m', \mathbf{R}\}},$$

where

$$\begin{cases} 0 \leq t \leq 5, & K = 0, \\ 6 \leq t \leq 11, & K = 1, \\ 12 \leq t \leq 17, & K = 2, \\ & \vdots \\ N - 6 \leq t \leq N - 1, & K = \frac{N}{6} - 1, \end{cases}$$

then

$$\Lambda(u_t) = \log \frac{\sum_m \sum_{m'} \gamma_1(\mathbf{R}_k, m', m) \alpha_{t-1}(m') \beta_t(m)}{\sum_m \sum_{m'} \gamma_0(\mathbf{R}_k, m', m) \alpha_{t-1}(m') \beta_t(m)},$$

where

$$\begin{aligned} \bullet \alpha_t(m) &= \sum_{a=0}^1 \sum_{m'=0}^{M-1} \gamma_a(\mathbf{R}_K, m', m), \\ \alpha_0(\mathbf{0}) &= 1, \alpha_0(m) = 0, m \neq 0. \\ \bullet \beta_t(m) &= \sum_{a=0}^1 \sum_{m'=0}^{M-1} \gamma_a(\mathbf{R}_{K+1}, m, m') \beta_{t+1}(m'), \\ \beta_N(\mathbf{0}) &= \frac{1}{M}, \beta_N(m) = 0, m \neq 0. \end{aligned}$$

The probabilities  $\gamma_a(\mathbf{R}_K, m', m)$ ,  $a = 0, 1$ , are determined from the transition probabilities of the time discrete Gaussian memoryless channel and from the state transition probabilities of the correspondent trellis as follows

$$\gamma_a(\mathbf{R}_K, m', m) = P\{S_t = m/S_{t-1} = m'\} p\{\mathbf{R}_K/u_t = a, S_t = m, S_{t-1} = m'\} P\{u_t = \frac{a}{S_t} = m, S_{t-1} = m'\}.$$

The transition probabilities  $P\{S_t = m/S_{t-1} = m'\}$  are defined from the *a priori* probabilities of the input bits. When they are equiprobable,  $P\{u_t = 1\} = P\{u_t = 0\} = \frac{1}{2}$  and we have  $P\{S_t = m/S_{t-1} = m'\} = \frac{1}{2}$ . The term  $p\{\mathbf{R}_K/u_t = a, S_t = m, S_{t-1} = m'\}$  is the transition probability of the AWGN channel and can be written as

$$\begin{aligned} p\{\mathbf{R}_K/u_t = a, S_t = m, S_{t-1} = m'\} &= \\ &= \sum_{iK} \sum_{jK} \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(r_{1K}-A_{xK})^2+(r_{2K}-B_{yK})^2}{2\sigma^2}\right] \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(r_{3K}-C_{sK})^2+(r_{4K}-D_{zK})^2}{2\sigma^2}\right], \end{aligned}$$

where  $xK$ ,  $yK$ ,  $sK$  and  $zK$  are the set of true hypotheses. The probability  $P\{u_t = a/S_t = m, S_{t-1} = m'\}$  is 0 or 1 because the convolutional encoder is a deterministic machine.

**Simulation Results.** For comparison purpose we consider the uncoded 16QAM modulation format with Gray code mapping in a channel with additive white Gaussian noise (AWGN), with average signal energy  $E_s = 10$  dB and variance  $\sigma^2 = 1,25 / 10^{(-E_b/N_0)/10}$ . The bit error rate (BER) *versus*  $E_b/N_0$ , characteristic for this 16QAM is shown in Fig. 4 (solid line). Now consider uncoded 2-12QAM modulation with 7 binary to two 12-ary symbol mapping in the same channel, with average signal energy  $E_s \approx 7,33$  dB and variance  $\sigma^2 \approx 0,52 / 10^{(-E_b/N_0)/10}$ . The BER *versus*  $E_b/N_0$ , characteristic for this 2-12QAM is shown in Fig. 4 (dotted line). The curves in Fig. 4 indicate for 2-12QAM an energy gain of about 2 dB, 3,1 dB and 3,8 dB for BER =  $10^{-3}$ ,  $10^{-4}$  and  $10^{-5}$ , respectively, operating with spectral efficiency 3.5 bits/symbol, i.e. with a rate loss less than 0,6 dB.

Now let us consider a coded 16QAM modulation format with Gray code mapping in a AWGN channel. The convolutional encoder has polynomial generator matrix given by  $G(D) = \begin{bmatrix} 1 & \frac{1+D+D^2+D^3+D^4}{1+D^4} \end{bmatrix}$  and the output symbols are punctured using the puncturing matrix given by  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ , the resultant code has rate  $R = \frac{3}{4}$  and the variance is  $\sigma^2 \approx 1,67 / 10^{(-E_b/N_0)/10}$ . The BCJR decoding algorithm is used. The bit error rate (BER) *versus*  $E_b/N_0$  characteristic for this system is shown in Figure 4 (dashed line). Now consider a coded 2-12QAM modulation with 7 binary to two 12-ary symbol mapping in the AWGN channel. The convolutional encoder has polynomial generator matrix given by  $G(D) = \begin{bmatrix} 1 & \frac{1+D^3+D^4+D^5+D^6}{1+D+D^3+D^4+D^6} \end{bmatrix}$  and the output symbols are punctured using the puncturing matrix given in (2), the resultant code has rate  $R = \frac{6}{7}$  and the variance is  $\sigma^2 \approx 0,61 / 10^{(-E_b/N_0)/10}$ . The BCJR algorithm is used as described before. The bit error rate (BER) *versus*  $E_b/N_0$  characteristic for this system is shown in Fig. 4 (dash-dot line).

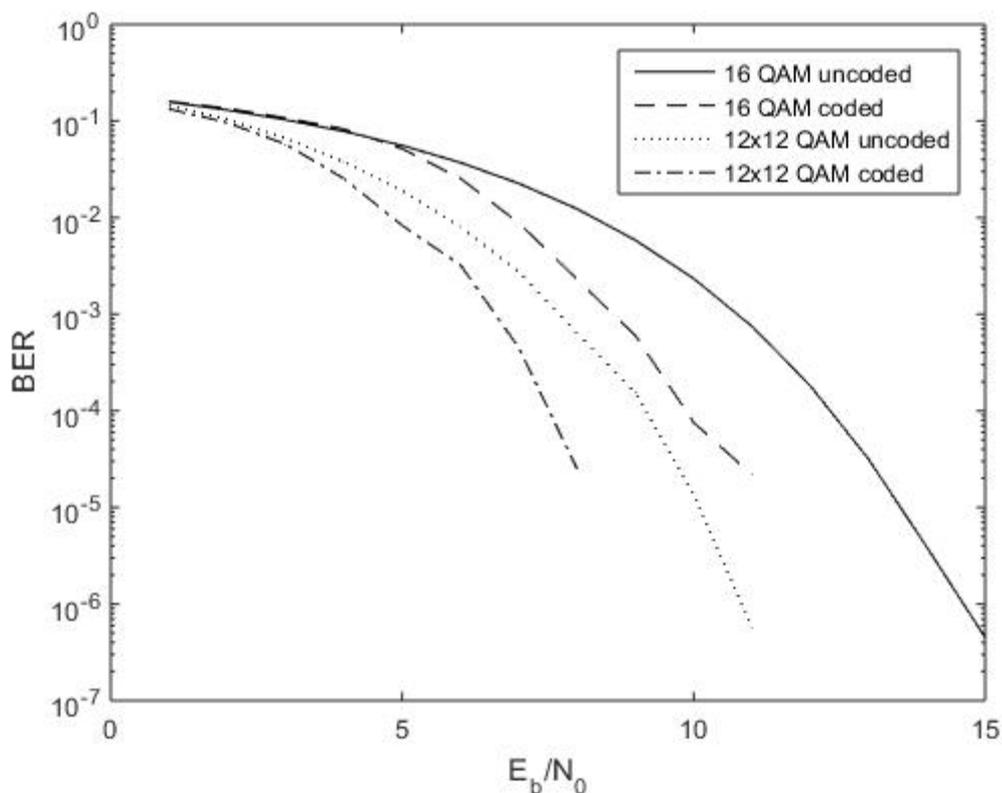


Figure 4 – BER versus  $E_b/N_0$  curves for 16QAM and 2-12QAM uncoded and coded

The curves in Fig. 4 indicate for 2-12QAM an energy gain of about 2 dB and 2,5 dB for BER =  $10^{-3}$  and  $10^{-4}$ , respectively. We can also observe that the system using 2-12QAM uncoded has an energy gain of about 0,8 dB when compared with the 16QAM coded.

**Conclusion.** In this Paper we propose a novel technique which allows a reduction of a power back up by introducing a 4-dimensional 2-12QAM modulation obtained by changes of 16QAM mapping. We show that the proposed technique allows at least a 2dB power efficiency without sacrificing spectral efficiency and additional hardware complexity.

#### ACKNOWLEDGMENTS

V. C. da Rocha Jr. acknowledges partial support by the Brazilian National Council for Scientific and Technological Development – CNPq, Project No. 307467/2015-5. Authors will to acknowledge help from Dr. Andrew Senior and Dr. Keith Barrat of Rinicom Ltd, who provided valuable comments on practical implementation of the proposed scheme.

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DOI 10.33243/2518-7139-2019-1-1-159-168