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DETERMINATION OF THE NECESSARY INDUCTOR CORE DIMENSIONS FOR SWITCHING ELECTRICAL ENERGY CONVERTERS

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ВИЗНАЧЕННЯ НЕОБХІДНИХ РОЗМІРІВ МАГНІТОПРОВОДУ ДРОСЕЛЯ ДЛЯ ІМПУЛЬСНИХ ПЕРЕТВОРЮВАЧІВ ЕЛЕКТРИЧНОЇ ЕНЕРГІЇ

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ОПРЕДЕЛЕНИЕ НЕОБХОДИМЫХ РАЗМЕРОВ МАГНИТОПРОВОДА ДРОССЕЛЯ ДЛЯ ИМПУЛЬСНЫХ ПРЕОБРАЗОВАТЕЛЕЙ ЭЛЕКТРИЧЕСКОЙ ЭНЕРГИИ

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Abstract. Each developer of switching electrical energy converters, sooner or later, faces the need to select or develop an inductor. However, currently, the method of choosing an inductor core for switching converters is very ambiguous, fragmented and confused. In the article, formulas are obtained that allow determining the minimum-necessary product of the core cross-sectional area by the core window cross-sectional area for the inductors of the most common switching converters circuits: buck, boost, buck-boost and fly-buck. In addition, the best mode of core operation, from the energy point of view, in which the maximum converters power density is provided both in the transmission and recuperation modes, is analyzed in the article. The article is intended for a wide range of electronics developers, including for specialists engaged in research of switching electrical energy converters.

Key words: electrical energy, switching converter/regulator, buck/boost/buck-boos/fly-back converter, inductor, core, cross-sectional area of the core/core window.

Анотація. Кожен розробник імпульсних перетворювачів електричної енергії рано чи пізно стикається з необхідністю вибору або розробки накопичувального дроселя. Однак на сьогоднішній день методика вибору магнітопроводів для цих приладів дуже неоднозначна, розрізнена і заплутана. Це призводить до того, що на практиці проектування або вибір дроселя для імпульсного перетворювача електричної енергії перетворюється в ітераційний процес, що полягає в переборі доступних магнітопроводів, з подальшою перевіркою чи задовольняє отриманий результат технічному завданню. Очевидно, що такий метод проектування вимагає значних витрат часу, а його результат може бути далекий від оптимального. У статті отримані формули, що дозволяють визначити мінімально-необхідний добуток площі поперечного перетину осердя на площу поперечного перетину вікна магнітопроводу для дроселів найбільш поширених схем імпульсних перетворювачів: зонижувальної, підвищувальної, інвертувальної і зворотноходової. Крім цього, у статті зроблено аналіз найкращого з енергетичної точки зору режиму роботи магнітопроводу, за якого забезпечується найбільша питома потужність перетворювачів електричної енергії як в режимі передачі, так і в режимі рекуперації,і на підставі проведеного аналізу визначено, що найбільшу потужність можна отримати при роботі магнітопроводу дроселя у граничному режимі. Розглянуто також вплив параметрів реальних магнітних матеріалів, зокрема остаточної намагніченості, на величину максимальної потужності перетворювача.

Стаття призначена для широкого кола розробників електроніки, в тому числі і для фахівців, що займаються дослідженнями імпульсних перетворювачів електричної енергії. Використання результатів проведених досліджень дозволяє значно скоротити витрати часу на проектування імпульсних перетворювачів електричної енергії як постійної, так і змінної напруги, а також підвищити їх головні техніко-економічні показники.

Ключові слова: електрична енергія, імпульсний перетворювач/регулятор, знижувальний/ підвищувальний/інвертувальний/зворотноходовий перетворювач, дросель, магнітопровід, площа поперечного перетину осердя/вікна.

Аннотация. Каждый разработчик импульсных преобразователей электрической энергии рано или поздно сталкивается с необходимостью выбора или разработки накопительного дросселя. Однако на сегодняшний день методика выбора магнитопровода для этих приборов очень неоднозначна, разрозненна и запутана. В статье получены формулы, позволяющие определить минимально-необходимое произведение площади поперечного сечения сердечника на площадь поперечного сечения окна магнитопровода для дросселей наиболее распространенных схем импульсных преобразователей: понижающей, повышающей, инвертирующей и обратноходовой. Кроме этого, в статье был выполнен анализ наилучшего с энергетической точки зрения режима работы магнитопровода, при котором обеспечивается наибольшая удельная мощность преобразователей электрической энергии как в режиме передачи, так и в режиме рекуперации. Статья предназначена для широкого круга разработчиков электроники, в том числе и для специалистов, занимающихся исследованиями импульсных преобразователей электрической энергии.

Ключевые слова: электрическая энергия, импульсный преобразователь/регулятор, понижающий/повышающий/инвертирующий/обратноходовой преобразователь, дроссель, магнитопровод, площадь поперечного сечения сердечника/окна.

At the present time, switching energy converters (SEC) are the basis of devices and power supply systems for modern radio and telecommunication equipment. Their technical and economic characteristics largely determine the quality of services provided by communication companies. A feature of switching conversion is the need to use components capable of accumulating a significant quantity of energy in electrical (capacitors) or magnetic (inductors) fields. But the power density of these components especially inductor is still low. Thus, the SECs have a significant weight and dimensions, which sometimes can exceed the weight and dimensions of other device units. Therefore, the search for ways to increase the power density of the SECs will actual for a long time.

One of the questions that any switching converters developer faces sooner or later is the choice or design of an inductor – the main component of the switching energy converter. In most cases, the inductor has maximum weight and dimensions. So, its miniaturization will always be first in the task list of the switching converters developer. However, despite the importance of the question, the method of choosing an inductor core, in contrast to the well-described algorithms for choosing cores for transformers, which are well described in the specialized literature, is very ambiguous, fragmented and confused. And this is despite the fact that the questions "Which magnetic core is necessary for the converter with such power?" or "What converter power can be developed on the basis of this magnetic core?" are a priority in calculating this component. In the end, in practice, the development of the inductor is an iterative process, consisting of sorting out the available standard cores, and then verifying whether the resulting device meets the technical specification. Obviously, such a design method requires considerable time and the result of development may be far from optimal.

This determined **the purpose of the article**, which consists the determining of the minimum-necessary inductor core dimensions for the most popular switching energy converters.

By analogy with transformers as the analyzed parameter the product $S_C S_W$ of the cross-sectional area of a core S_C and cross-sectional area of acorn window S_W was chosen. Also, for the task solution, the analysis method proposed in [1] was used. According to [1], there is a switching regulator (SR) in the SEC that converts electric energy. Depending on the variant of the connection the input and output of the SR to the input and output of the SEC (Fig. 1, *a*), the power of the regulator P_{SR} can be less than the power of the converter P_{SEC}. For example, when the input voltage

U_{IN} of a buck converter is twice as high as its output voltage U_{OUT} (U_{OUT}/U_{IN} = 0,5), the power of the SR is only half the power of the converter (P_{SR}/P_{SEC} = 0,5) (Fig. 1, *b*), which means that an inductor with a smaller power capacity can be used in this converter.

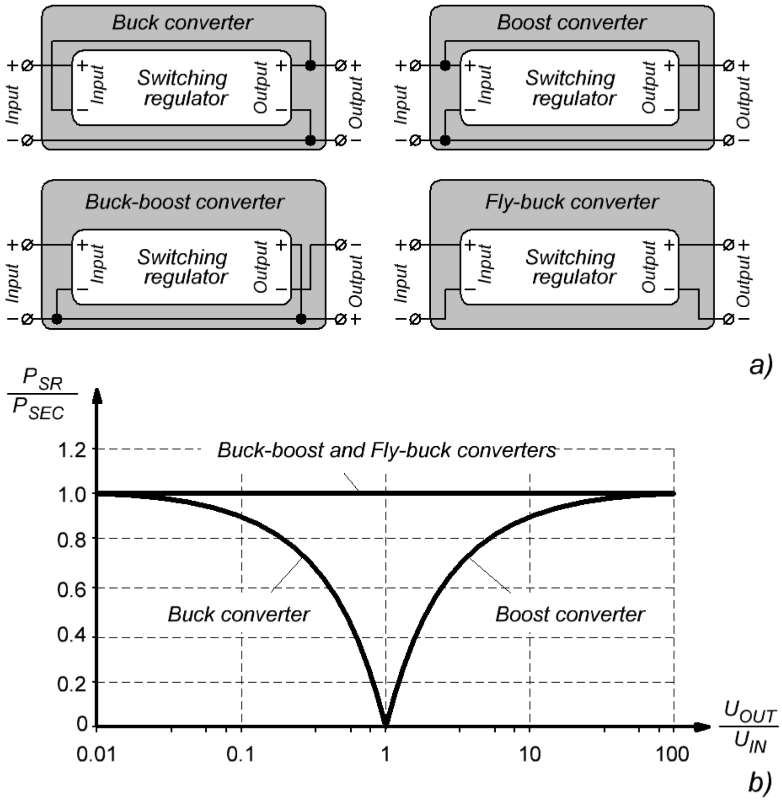

Figure 1 – The block diagrams of the most popular SECs *a*) and the dependence of their SR's relative power on the ratio of the SEC's input and output voltages *b*)

Also, according to [1], it is necessary to consider a fly-buck converter as a switching regulator, since the well-known buck, boost and buck-boost converters can be obtained from this scheme. This allowed using the fly-back converter as a basis for subsequent analysis.

In the simplest case, the process of switching conversion consists of two stages (Fig. 2). At the first stage, there is an exchange of energy W_{PULSE} between the capacitor *C1* and the inductor *L1*. At the second stage, there is an exchange of energy W_{PULSE} between the inductor *L1* and the capacitor *C2*. Thus, the inductor *L1* is intermediate energy storage and it must be able to hold the energy W_{PULSE} in the magnetic field of the core.

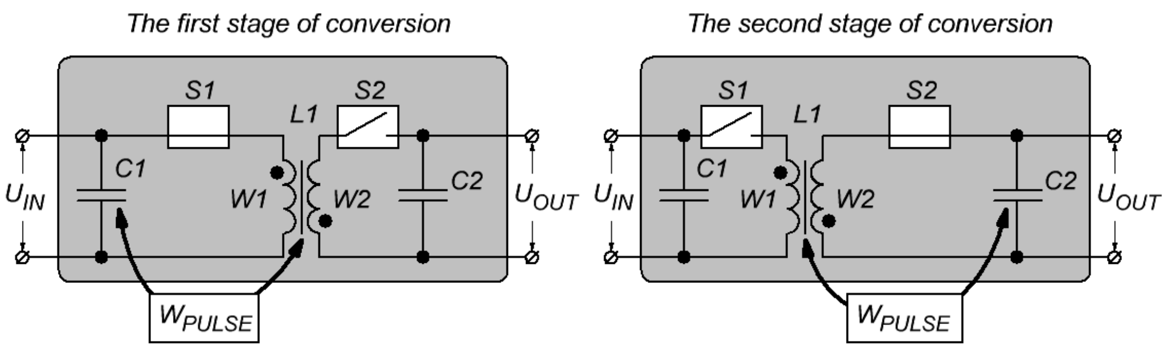

Figure 2 – The principle of switching conversion

The necessary value W_{PULSE} can be calculated based on the switching frequency f and the maximum power of the switching regulator P_{SR} which equal the maximum speed of energy transfer through the magnetic field of the inductor core [1]:

$$W_{PULSE} = \frac{P_{SR}}{f}. \tag{1}$$

According to the analysis of magnetic processes in switching converter inductors [2], the quantity of energy accumulated in a magnetic field can be determined based on the parameters of the magnetic flux of the inductor core:

$$W_{PULSE} = \frac{\Delta\Phi_1 \Phi_{AVR1}}{A_L}, \tag{2}$$

where Φ_{AVR1}, $\Delta\Phi_1$ are the average value and the change of the magnetic flux at the first conversion stage, respectively; A_L is the reference parameter of the core (inductance per turn), used in calculating winding inductances L:

$$L = N^2 A_L, \tag{3}$$

where N is number of winding turns.

The P_{SR} and f values are known at the design stage, but the operation mode of the inductor core, on which the product $\Delta\Phi_1\Phi_{AVR1}$ depends, the developer need to choose. Analysis of the possible modes of inductor core operation is needed in order to find the conditions for the maximum value of the converted power P_{SR} at the same value of A_L.

In general case, switching energy converters can operate in four different modes (Fig. 3): continuous conduction mode, boundary mode, discontinuous conduction mode, and forced continuous conduction mode. Each mode has advantages and disadvantages and is characterized by different $\Delta\Phi_1$ and Φ_{AVR1} values at the same value of the P_{SR}. Therefore, a different volume of magnetic core is required for different modes.

The $\Delta\Phi_1$ and Φ_{AVR1} values are interrelated and limited by the maximum permissible value of the magnetic flux Φ_{SAT} at which saturation of the core occurs:

$$\begin{cases} \left| \Phi_{AVR1} + \dfrac{|\Delta\Phi_1|}{2} \right| \le \Phi_{SAT} \\ |\Delta\Phi_1| \le 2\Phi_{SAT} \end{cases}. \tag{4}$$

In practice, the magnetic flux ripple amplitude is usually specified or chosen by the developer depending on the core losses. Determining Φ_{AVR1} from (4) as a function of $\Delta\Phi_1/\Phi_{SAT}$, it is possible to determine from (2) the maximum possible energy quantity W_{PULSE}, which can be stored in the magnetic core, at the specific $\Delta\Phi_1/\Phi_{SAT}$ value (Fig. 4) [2].

Fig. 4 shows that the most effective mode of the inductor operation from the energy point of view is the mode when the peak-to-peak magnetic flux ripple amplitude $|\Delta\Phi_1|$ is equal to the saturation flux Φ_{SAT} ($\Delta\Phi_1/\Phi_{SAT} = \pm 1$). In this case, the maximum energy quantity is transferred through the inductor magnetic field, and, consequently, the switching regulator power P_{SR} is maximum both in the transmission (when $\Delta\Phi_1/\Phi_{SAT} > 0$) and recuperation modes (when $\Delta\Phi_1/\Phi_{SAT} < 0$). According to the Fig. 3, it occurs when the SEC operates in the boundary or discontinuous modes. However, in the discontinuous mode the converter has the pauses between conversion cycles, because $t_1 + t_2 < T$. According to (1), it is equivalent to decreasing the switching frequency f while energy conversion quantity W_{PULSE} is fixed. Therefore, it is equivalent to decreasing the P_{SR}. Thus, the most effective mode, when the maximum energy quantity is transferred through the inductor magnetic field, is the boundary mode.

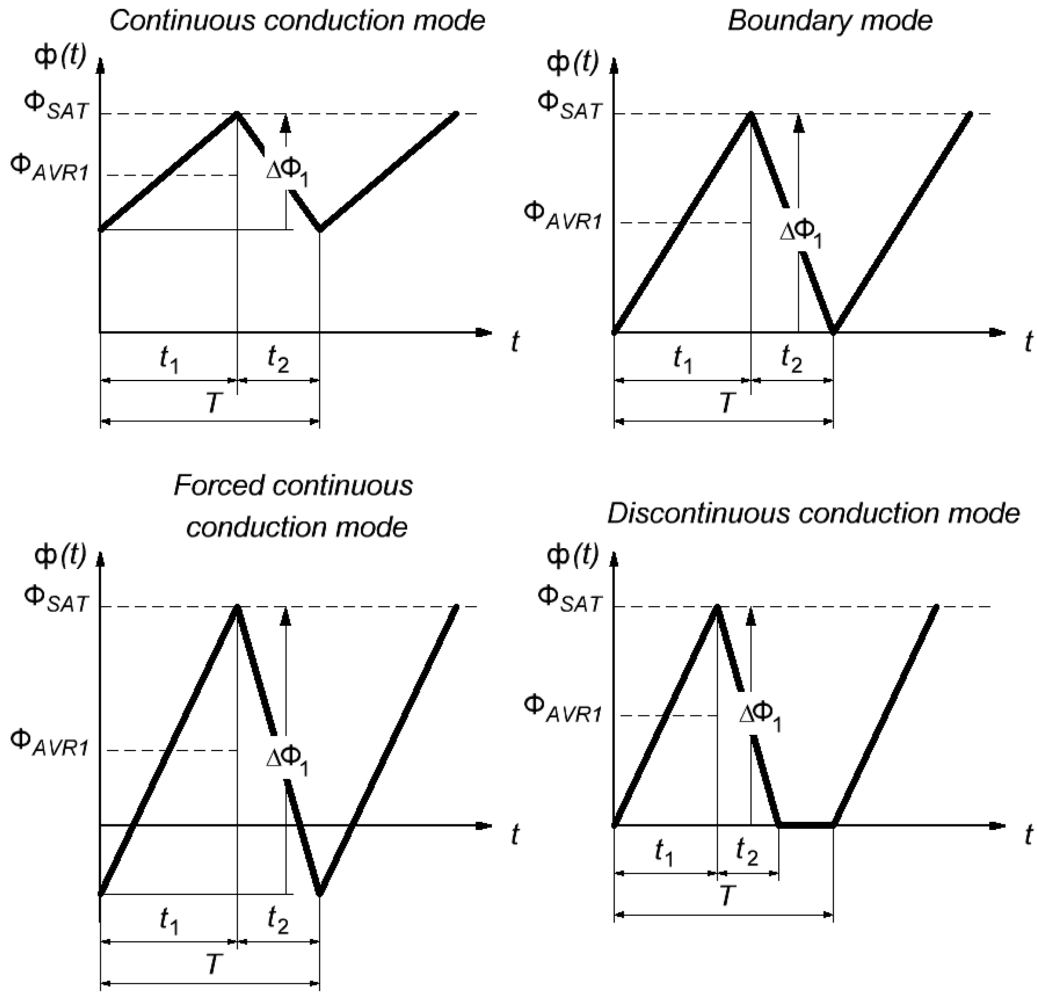

Figure 3 – Magnetic flux of the inductor core ϕ(t) in various operating modes

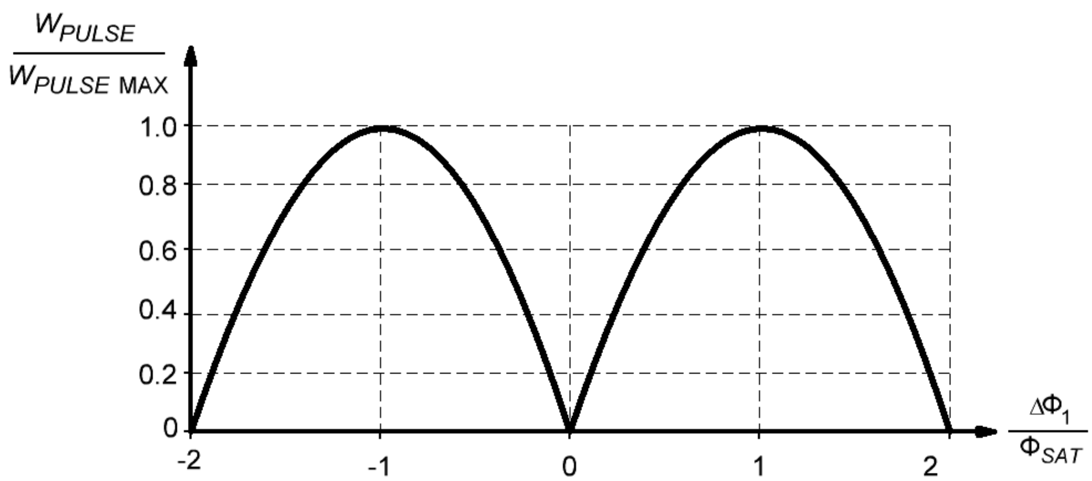

Figure 4 – Dependence of the quantity of energy W_{PULSE} on the value of magnetic flux change $\Delta\Phi_1$

The theoretical maximum of the switching regulator power can be achieved when its inductor operates under the condition $|\Delta\Phi_1| = \Phi_{SAT}$. However, in practice, the real $|\Delta\Phi_1|$ value does not reach the theoretical maximum because the real ferromagnetic materials have a remanence flux density and the real converters have a power reserve. Thus, the real $|\Delta\Phi_1|$ value is less than the theoretical maximum and equal to:

$$\left|\Delta\Phi_1\right| = S_{MM}(B_{MAX} - B_R), \tag{5}$$

where B_{MAX} is the maximum flux density chosen by the developer; B_R is the remanence flux density and S_{MM} is the cross-sectional magnetic material area.

When the inductor operates in the mode defined by formula (5), its core is magnetized by a certain particular cycle. In this case, inductor core magnetic flux diagrams are shown in the Fig. 5. In according to (2), the SEC has to operate in the continuous conduction mode. However, in practice, the remanence flux density B_R influence on the energy processes can be neglected. Thus, the SEC actually operates in the boundary mode, and we can write:

$$\Phi_{AVR1} \approx 0,5\Delta\Phi_1. \tag{6}$$

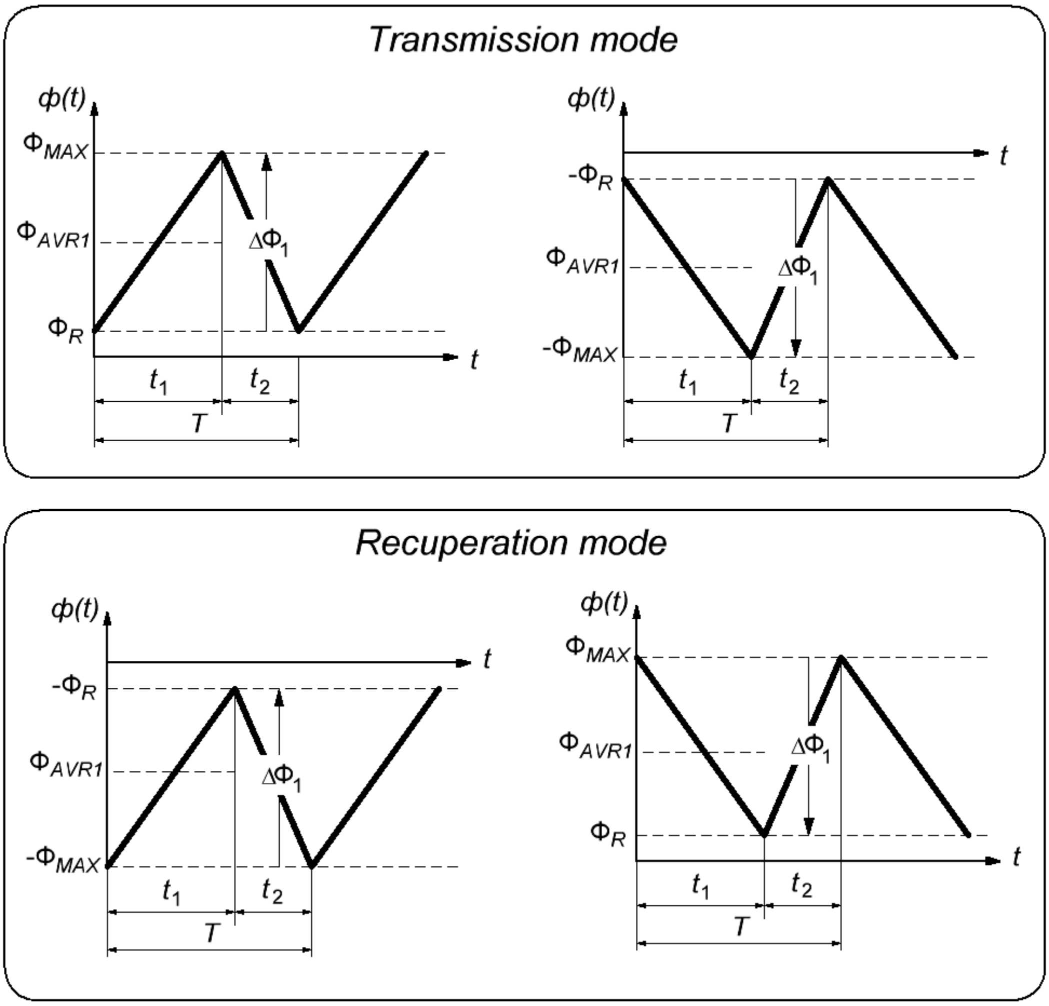

Figure 5 – Inductor core magnetic flux at the maximum power mode

Substituting formulas (5) and (6) in (2) and equating the result to (1), we can obtain the minimally necessary cross-section area of the inductor core S_C:

$$S_C = \frac{1}{k_C (B_{MAX} - B_R)} \sqrt{\frac{2P_{SW} A_L}{f}}\,, \tag{7}$$

where $k_C = S_{MM}/S_C$ is the fill factor of core cross-section area with a magnetic material.

It is important to determine the minimally necessary cross-section area of the core window S_W. In general case, the fly-back converters inductor contains two windings (Fig. 2), which must be placed in the core window. At the first stage of the conversion with the duration t_1, the winding *W1* is active and the winding *W2* is not used. At the second stage of the conversion with duration t_2, on the contrary, the winding *W1* is not used and the current flows only in the winding *W2*.

The minimum area required to place windings *W1* and *W2* in the core window is determined by formulas:

$$S_1 = \frac{N_1 I_{RMS1}}{J_1}, \quad S_2 = \frac{N_2 I_{RMS2}}{J_2}\,, \tag{8}$$

where S_1, S_2 are the area occupied by the conductor; N_1, N_2 are the number of turns; I_{RMS1}, I_{RMS2} are the root mean square (RMS) values of the currents and J_1, J_2 are the current density, respectively, of the windings *W1* and *W2*.

When the SEC operates in the boundary mode the inductor windings effective currents values can be determined on the basis of an electrical processes analysis performed in [3]:

$$I_{RMS1} = \sqrt{k_1 \frac{I_{m1}^2}{3}}\,, \qquad I_{RMS2} = \sqrt{k_2 \frac{I_{m2}^2}{3}}\,, \tag{9}$$

where $I_{m1} = |\Delta I_1|$, $I_{m2} = |\Delta I_2|$ are the peak-to-peak amplitude of the current ripple in the windings *W1* and *W2*, respectively; $k_1 = t_1/T$, $k_2 = t_2/T$ are the relative duration of the first and second conversion stages, respectively (Fig. 6).

When there are not conversion losses, the SR's average input $I_{IN\,SR}$ and output $I_{OUT\,SR}$ currents can be expressed on base of the SR's power P_{SR} and input $U_{IN\,SR}$ and output $U_{OUT\,SR}$ voltages (Table 1):

$$I_{IN\ SR} = P_{SR} / U_{OUT\ SR}\,, \quad I_{OUT\ SR} = P_{SR} / U_{OUT\ SR}\,. \tag{10}$$

Table 1 –The SR's input and output voltages for different SEC [2]

SEC	$U_{IN\,SR}$	$U_{OUT\,SR}$
Buck	$U_{IN} - U_{OUT}$	U_{OUT}
Boost	U_{IN}	$U_{OUT} - U_{IN}$
Buck-boost	U_{IN}	U_{OUT}
Fly-buck	U_{IN}	U_{OUT}

A feature of the fly-back converter is the equality of the average values of the SR's input and output current, corresponding to the currents of the inductor windings *W1* and *W2*:

$$I_{AVR1} = I_{IN\ SR}\,; \quad I_{AVR2} = I_{OUT\ SR}\,. \tag{11}$$

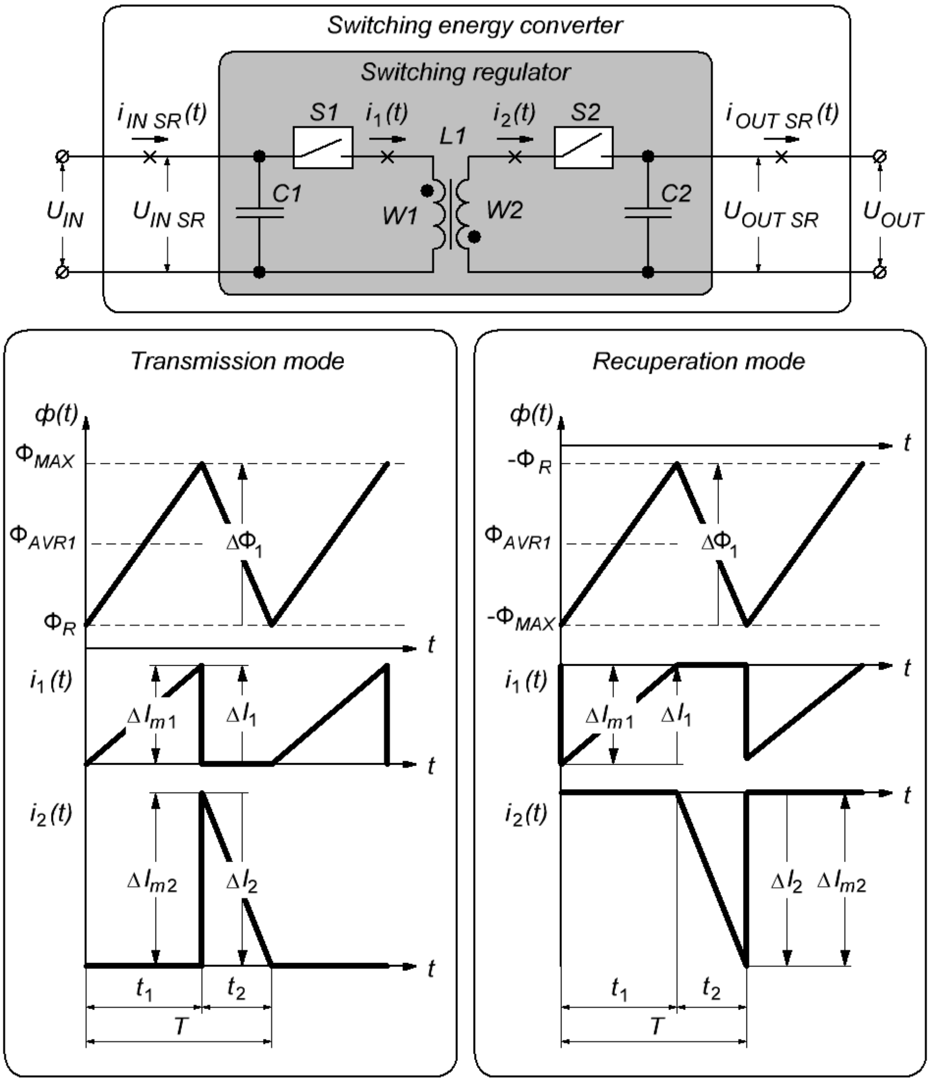

Figure 6 – SR’s electrical processes at the boundary operation mode

In addition, the winding current average values can also be expressed on base of the inductor current parameters [3]:

$$I_{AVR1} = k_1 \frac{I_{m1}}{2}; \quad I_{AVR2} = k_2 \frac{I_{m2}}{2}. \tag{12}$$

Expressing the peak-to-peak amplitude of the inductor current ripple values I_{m1} and I_{m2} from (11) with considering (10) and (12), and substituting for the result into (9), we obtain a formula relating the winding current values to the SR parameters:

$$I_{RMS1} = \frac{2P_{SR}}{U_{IN\ SR}\sqrt{3k_1}}; \quad I_{PMS2} = \frac{2P_{SR}}{U_{OUT\ SR}\sqrt{3k_2}}. \tag{13}$$

The windings current changes occur with the voltages $U_{IN\ SR}$ and $U_{OUT\ SR}$ connected to windings *W1* and *W2* through the open switches *S1* and *S2* at the first and second conversion stages, respectively. It allows determine the winding current ripples values by the formulas [3]:

$$I_{m1} = \frac{U_{IN\ SR}}{L_1} k_1 T\ ; \quad I_{m2} = \frac{U_{IN\ SR}}{L_2} k_2 T\ , \tag{14}$$

where L_1, L_2 are the inductances of the windings *W1* and *W2* respectively.

Expressing the peak-to-peak amplitude of the inductor current ripple values I_{m1} and I_{m2} from (11) with considering (10) and (12), and equating the obtained relation with (14), we can, taking into consideration (3), obtain the formula relating the inductor windings number of turns with the SR parameters:

$$N_1 = \frac{U_{IN\ SR} k_1}{\sqrt{2 A_L P_{SR} f}}\ ; \qquad N_2 = \frac{U_{OUT\ SR} k_2}{\sqrt{2 A_L P_{SR} f}}\ . \tag{15}$$

Substituting (13) and (15) into (8), we can obtain the area necessary for placing the conductive material of the inductor windings:

$$S_1 = \frac{1}{J_1}\sqrt{\frac{2 P_{SR} k_1}{3 A_L f}}\ , \qquad S_2 = \frac{1}{J_2}\sqrt{\frac{2 P_{SW} k_2}{3 A_L f}}\ . \tag{16}$$

Assuming the current density of the windings *W1* and *W2* to be the same ($J_1 = J_2 = J$), and considering the presence of the core coil bobbin, isolation, non-ideality of winding and other manufacturing features, we can obtain the required window area:

$$S_W = S_1 + S_2 = \frac{1}{k_W J}\sqrt{\frac{2 P_{SW}}{3 A_L f}}\left(\sqrt{k_1} + \sqrt{k_2}\right), \tag{17}$$

where k_W is the fill factor of core window cross-section area with a conductor material.

The relative durations of the conversion stages are determined by the voltages at the input and output of the SR, it allows writing [1]:

$$k_1 = \frac{\bar{U}_{OUT\ SR}}{n_{21} + \bar{U}_{OUT\ SR}}\ ; \qquad k_2 = \frac{n_{21}}{n_{21} + \bar{U}_{OUT\ SR}}\ , \tag{18}$$

where $\bar{U}_{OUT\ SW} = U_{OUT\ SW} / U_{IN\ SW}$ is the SR's relative output voltage; $n_{21} = N_2/N_1$ is winding turns ratio of the inductor (inductor turns ratio).

Substituting (18) into (17) and multiplying the result by (7), we obtain the required product of the cross-sectional area of the core by the cross-sectional area of the core window:

$$S_C S_W = \frac{2 P_{SR}}{\sqrt{3} k_C k_W J (B_{MAX} - B_R) f}\left(\sqrt{k_1} + \sqrt{k_2}\right);$$

$$S_C S_W = \frac{2 P_{SR}}{\sqrt{3} k_C k_W J (B_{MAX} - B_R) f}\left(\sqrt{\frac{\bar{U}_{OUT\ SR}}{n_{21} + \bar{U}_{OUT\ SR}}} + \sqrt{\frac{n_{21}}{n_{21} + \bar{U}_{OUT\ SR}}}\right). \tag{19}$$

Analyzing (19), it can be seen that the dimensions of the inductor core dependent on the conventional inductive elements parameters (the converter power P_{SR}, the magnetic flux parameters, the windings current density, the core and window fill factors), and the SEC-specific components (k_1 and k_2)which are depended with operating mode. It can be explained with the nonlinear form of the windings current (Fig. 6), which leads to increasing the currents' effective values and the conductor additional heating.

For example, in the boundary mode $I_{RMS1(2)} = 2I_{AVR1(2)} / \sqrt{3k_{1(2)}}$ [3]. With equal durations of the conversion stages ($k_1 = k_2 = 0{,}5$) the effective values of the winding currents are 1,63 times higher than their average values. And it, accordingly, requires an increase in the core window of the inductor.

In addition, it should be noted that the overall dimensions do not directly depend on the type of magnetic material and core design features. In formula (19) only the ripple amplitude of flux density ($B_{MAX} - B_R$) depends on the magnetic material.

Conclusion. Any switching energy converter is the result of a compromise between the dimensions, cost and efficiency of the final device, which the developer takes on the basis of specific technical specifications. With formulas (19), it can be possible to determine the minimum-necessary size of the inductor core, but it does not guarantee that the SEC created on its basis will have high technical characteristics. For example, the choosing of the SEC's operating point in the continuous conduction mode area makes it possible to reduce the ripple amplitude of flux density and reduce the inductor core losses. However, this will lead to the fact that the core with larger product S_{CW} than product calculated by the formula (19) need to use.

Thus, formulas (19) allow only a preliminary assessment of the suitability of the core for specific technical specifications. In order to obtain the final result, as in the transformers calculation, it is necessary to make a detailed inductor calculation. However, despite it, the obtained formulas will be useful both for a wide range of electronics developers and for specialists engaged in research of switching electrical energy converters. Also, if it necessary, the method of determining $S_C S_W$, proposed in the article, will make it possible to easily obtain analogous relations with other initial conditions.

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