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# COMPOSITE WALSCH-BARKER SEQUENCES

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**Abstract.** In the article methods of forming and processing of Walsh-Barker composite sequences are developed. The relevance of using Barker codes as generating sequences of a composite code intended for generating a synchronization signal is shown, without limitations on the choice of the shape of elementary symbols. The possibility of synthesizing composite sequences with given properties by an adequate choice of elementary symbols and generating sequences is investigated. It is shown that the use of Walsh's orthogonal sequences as elementary ones, introduces the self-synchronization property in the composite Walsh-Barker sequences and a number of new beneficial properties. The orthogonality property of the Walsh sequences and the orthogonality properties of the composite sequences can be simultaneously transmitted on a single carrier of several composite signals, modulated by information symbols. To allocate transmitted information symbols, the matched filtering of the transferred sum is used. The article is followed by diagrams obtained by simulating processes and sequences using the Hewlett Packard HPVEE Visual Object-Oriented Programming Package.The possibility of simultaneous and independent transmission of various modulated composite sequences are proved. The possibilities of the practical application of composite sequences in perspective telecommunication systems are estimated.

**Key words:** Walsch-Barker composite sequences, generating/detecting methods.

**Анотація.** У статті розроблені методи формування та обробки композитних послідовностей Уолша-Баркера. Показана актуальність застосування кодів Баряк утворюючих послідовностей композитного коду, призначеного для формування сигналу синхронізації, без обмежень на вибір форми елементарних символів. Досліджено можливість синтезу композитних послідовностей із заданими властивостями шляхом адекватного вибору елементарних символів і утворюючих послідовностей. Показано, що використання ортогональних послідовностей Уолша як елементарних вносить у композитні послідовності Уолша-Баркера властивість самосинхронизації й низку нових корисних властивостей. Досліджена властивість ортогональності послідовностей Уолша і витікаюча з цього властивість ортогональності композитних послідовностей допускають одночасну передачу на одній несучій декількох композитних сигналів, модульованих інформаційними символами. Для виділення переданих інформаційних символів використовується узгоджена фільтрація переданої суми. Стаття супроводжується демонстрацією діаграм, отриманих в результаті моделювання процесів і послідовностей засобами пакета візуального об'єктно-орієнтованого програмування HPVEE фірми Hewlett Packard. Теоретично й експериментально доведена можливість одночасної і незалежної передачі різних модульованих композитних послідовностей. Оцінені можливості практичного застосування композитних послідовностей в перспективних телекомунікаційних системах.

**Ключові слова:** коди Баркера, утворюючі послідовності, послідовності Уолша, матриця Адамара, алгоритм Сильвестра, композитні послідовності Уолша-Баркера, самосинхронізація, методи формування/обробки, узгоджений фільтр, автокореляційна функція, взаємокореляційна функція, імпульсна характеристика фільтра, ортогональність послідовностей, кратність передачі.

**Аннотация.** В статье разработаны методы формирования и обработки композитных последовательностей Уолша-Баркера. Показана актуальность применения кодов Баркера в качестве образующих последовательностей композитного кода, предназначенного для формирования сигнала синхронизации, без ограничений на выбор формы элементарных символов. Исследована возможность синтеза композитных последовательностей с заданными свойствами путем адекватного выбора элементарных символов и образующих последовательностей. Показано, что использование ортогональных последовательностей Уолша в качестве элементарных вносит в композитные последовательности Уолша-Баркера ряд новых полезных свойств. Проведено компьютерное моделирование и оценка возможностей практического применения композитных последовательностей в перспективных телекоммуникационных системах.

**Ключевые слова:** композитные последовательности, методы формирования/обработки.

**Introduction.** The forming method for Barker's composite sequences (codes) was first described in [1]. In accordance with this method, the composite code was composed of so-called *elementary* symbols (or sequences of symbols) according to the rules of the *generating* sequence, which coincides with the Barker code. Moreover, the use of Barker sequences guaranteed *unique properties* of the composite code intended for the formation of the synchronization signal (the so-called *self-synchronization property*). Further analysis of this method made it possible to *generalize* the rules for the forming of Barker *composite sequences* (codes) in the order of development of the ideas of [1], which reduce to the following:

1. The use of Barker codes as generating sequences guarantees the *self-synchronization property* of the composite code,
2. The choice of the form of *elementary* symbols (or sequences of symbols) is not superimposed. In particular, in the original Barker`s work [2], as well as in further manuals [3-5], the symbols (+1) are used as elementary ones, and
3. The properties of *elementary* symbols and generating sequences somehow "appear" in the properties of the composite sequence.

This opens up possibilities for the synthesis ("*composition*") of composite sequences with given properties by an adequate choice of *elementary* symbols and *generating* sequences. In particular, the most successful is the use of orthogonal Walsh sequences [6] as elementary ones. The Walsh-Barker composite sequences obtained in this way not only *preserve the orthogonality properties* of the original Walsh sequences (widely used in CDMA mobile communication systems [6]), but also acquire *additional* self-synchronization properties and a number of *new useful* properties. ***The goal of this article*** is to develop methods for the formation/processing of Walsh-Barker composite sequences, to study the properties and to assess the possibilities of using these sequences in advanced telecommunication systems. The article contains diagrams obtained as a result of modeling processes and sequences using the software of visual object-oriented programming – Hewlett Packard HPVEE.

**Orthogonal walsh sequences.** The basis for constructing orthogonal Walsh sequences is the following procedure [6]. We use a convenient and compact matrix form. Suppose we are given a $(2 \times 2)$ Hadamard matrix

$$\mathbf{H}_2 = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} . \qquad (1)$$

We use the Sylvester algorithm to obtain a matrix of twice the size

$$\mathbf{H}_4 = \begin{vmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{vmatrix} . \tag{2}$$

Continuing this procedure, we obtain a set of Walsh sequences of length $m = 8$ in which the symbols of ones with signs are replaced by their signs

$$\mathbf{W}_8 = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \end{bmatrix} = \begin{bmatrix} + & + & + & + & + & + & + & + \\ + & - & + & - & + & - & + & - \\ + & + & - & - & + & + & - & - \\ + & + & + & + & - & - & - & - \\ + & - & - & + & + & - & - & + \\ + & + & + & + & - & - & - & - \\ + & - & + & - & - & + & - & + \\ + & - & - & + & - & + & + & - \end{bmatrix}. \tag{3}$$

The rows of the matrix (3) are *mutually orthogonal.* According to the author of the monograph [6], by continuation of this procedure one can obtain ensembles of Walsh functions of sufficiently large size $m$ (where $m$ is an integer power of 2). In the following, Walsh sequences with length of $m = 8$ characters will be used, written as row matrices:

$$w_0 = [+ \quad + \quad + \quad + \quad + \quad + \quad + \quad +], \tag{4}$$

$$w_1 = [+ \quad - \quad + \quad - \quad + \quad - \quad + \quad -]. \tag{5}$$

**Barker sequences (codes).** According to the data in [1, Table 1], to date, a limited number of short-length Barker codes are known $M < 13$. In order to reduce the volume of the article, we give the form of the Barker code matrix of a moderate length $M = 7$, which is enough to illustrate the method described:

$$C7 = [+ \quad - \quad + \quad + \quad - \quad - \quad -].$$

Or, in a formal record

$$C7 = [c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5 \quad c_6 \quad c_7]. \tag{6}$$

The form of the Barker sequence is shown in Fig. 1.

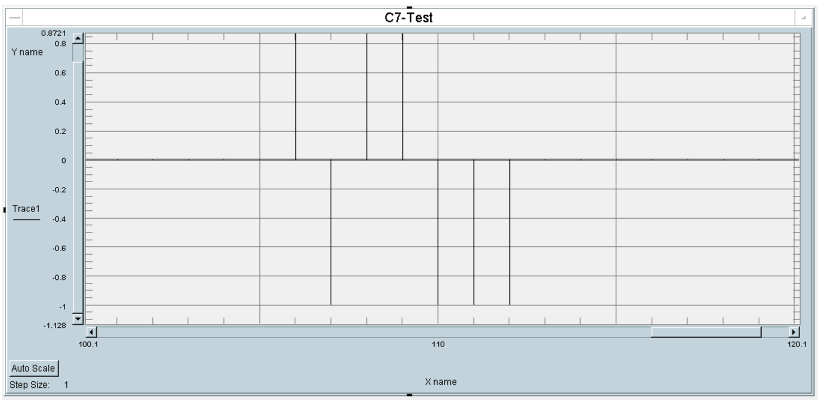


Figure 1 – Test Barker sequence $C7$

**Forming of Walsh-Barker composite sequences.** A matrix recording ensures the availability of a compact representation of the forming method. The composite sequence is determined by the prod*uct of the matrices* of the selected *elementary* Walsh sequences (4) or (5) by the matrix of the Barker generating sequence (6) (below we use the compact record proposed in [1])

$$[C7/w_i]=[w_i][C7], \tag{7}$$

where $i$ = (0…1) – number of the selected elementary sequence.

Since the Barker sequence (6) contains a set of constant numbers, the sequence (7) can be represented as follows:

$$[C7/w_i]=[w_ic_1 \quad w_ic_2 \quad w_ic_3 \quad w_ic_4 \quad w_ic_5 \quad w_ic_6 \quad w_ic_7]. \tag{8}$$

The record (8) defines simple ways of software or hardware implementation of the forming method: the selected elementary sequence is fed to the input of a chain of sequentially connected shift registers with outputs added after multiplying by the weigh coefficients of the generating sequence (6). Fig. 2 shows the timing diagram of the composite sequence $C7/w_0$ formed in this way.

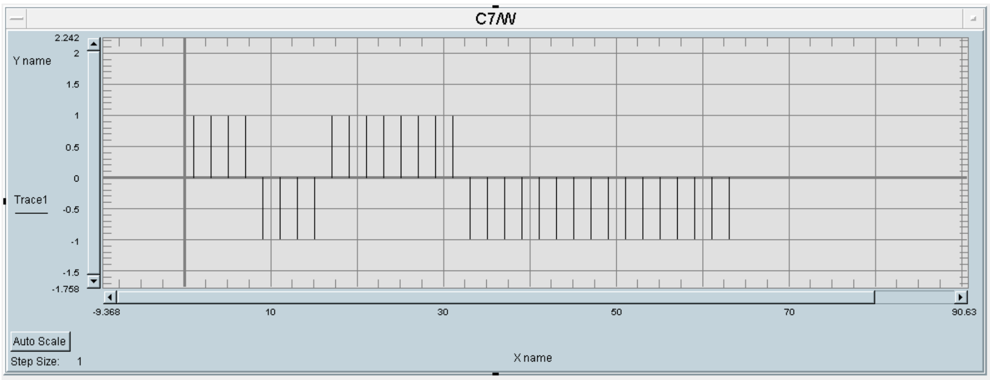


Figure 2 – Time Diagram of the sequence $C7/w_0$

**Processing composite sequences.** Separation of synchronizing Barker pulses from the digital data stream is performed using matched filters (MF). In accordance with the theory of signal processing, the *impulse response* of a filter matched to a discrete signal $C(t)$ to an accuracy of $a$ coincides with the mirror image of the waveform:

$$h(t)=aC(t_0-t), \tag{9}$$

where $t_0$ is MF output sampling moment.

When processing sequences, the concept of a *filter matched with the sequence* is also applicable. In this case, if a sequence is specified in the form of a row matrix (6), the impulse response of the matched filter is a *mirror image* of the processed sequence. In this case, the impulse response of the matched filter will be

$$[h(C7)]=[h_1=c_7 \; h_2=c_6 \; h_3=c_5 \; h_4=c_4 \; h_5=c_3 \; h_6=c_2 \; h_7=c_1]. \tag{10}$$

The result of processing the sequence $C7$ by a matched filter with an impulse response (10) is shown in Fig. 3. According with theory [1], the result of the processing coincides with the aperiodic autocorrelation function (ACF) of such a sequence:

$$R(k)=\frac{1}{M}\sum_{i=0}^{M}c(i)c(i+k), \tag{11}$$

and $R(k)=M$ for $k=0$; $R(k)=0$ for $k>1$, $(m-k)$ is even; $R(k)=\pm(1/M)$ for $k>1$.

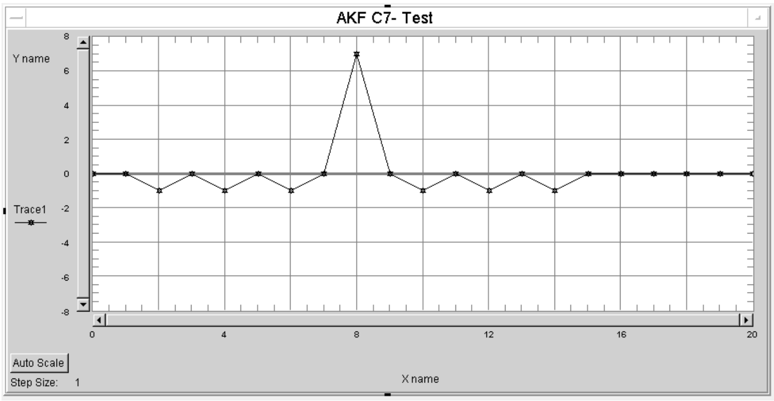


Figure 3 – The result of processing the Barker *C*7 sequence with a matched filter

In the time diagram, a "spike" of the autocorrelation function is seen, whose value in accordance with (11) is $R(k)=M$ for $k=0$. The unique form of ACF of composite sequence can be used not only for synchronization, but also for the *transmission of digital information*. For this, an amplitude modulation of the composite signal by the information symbol *U* should be used. The transmitted signal will be $[UC7/w_i]$. Fig. 4 shows the results of modeling the process of amplitude modulating of a composite signal by a linearly increasing modulating symbol *U* = (1,2,3,…,5,6,7,...). Such a method of *multilevel* information transfer is characterized in that the position of the ACF spike on the time axis can be used to estimate the moment of counting the value of the spike's amplitude (those. for *time synchronization*, where, the ACF *spike amplitude contains information about the transmitted symbol)*. If the energy of each symbol of the composite sequence is $E_s$ , then, in accordance with the formula (11), in the formation of the ACF spike of the composite signal with the length *m* of the elementary sequence and the length of the generating sequence *M*, the amplitude of the ACF spike rises to the value of

$$R(k)_{\max}=MmE_s \text{ for } k=0\,. \tag{12}$$

This is due to the effect of composite sequence symbols *energy accumulation* in forming of ACF and provides an increase in noise immunity of the ACF spike. To obtain a high noise immunity for the received signal registration, it is necessary to provide a large *difference in the levels of ACE spikes in comparison with the levels of the ACF side lobes*. Fig. 3 and 4 confirm this *important* property of the composite signal.

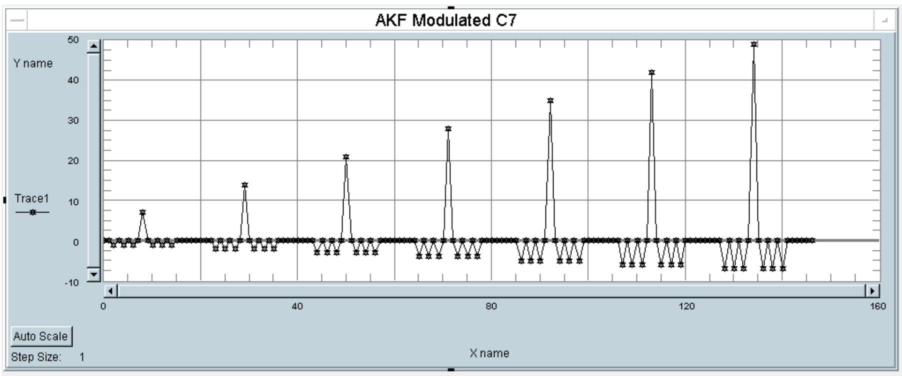


Figure 4 – Amplitude modulation of the composite signal ACF shape

Determing the impulse response of the filters matched to the above-mentioned elementary sequences (4) and (5), we find:

$$[h(w_0)] = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1], \; [h(w_1)] = [-1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1]. \tag{13}$$

An important role in the theory of processing digital sequences is played by the *correlation properties* [6]:

– ***Autocorrelation function*** (ACF) of the sequence $w_i$, calculated as the scalar product of the sequence $w_i$ with the impulse response of the filter matched to it:

$$[R(w_i, w_i)] = [w_i]\,[h(w_i)]; \tag{14}$$

– ***Cross-correlation function*** (CCF) of different sequences $w_i$ and $w_j$, calculated as the scalar product of the sequence $w_i$ with the impulse response of the filter matched to the sequence $w_j$:

$$[R(w_i, w_j)] = [w_i]\,[h(w_j)]. \tag{15}$$

A universal VEE program has been developed for calculating the correlation functions (14) and (15). The results are shown in Fig. 5 and 6. Here, the dependencies of the matched filters outputs (ACF and CCF, respectively) are shown when different sequences are fed to the MF inputs. Next to the time diagrams there is a table of ACF and CCF values for the selected type of elementary sequence, depending on the current time values (left column).

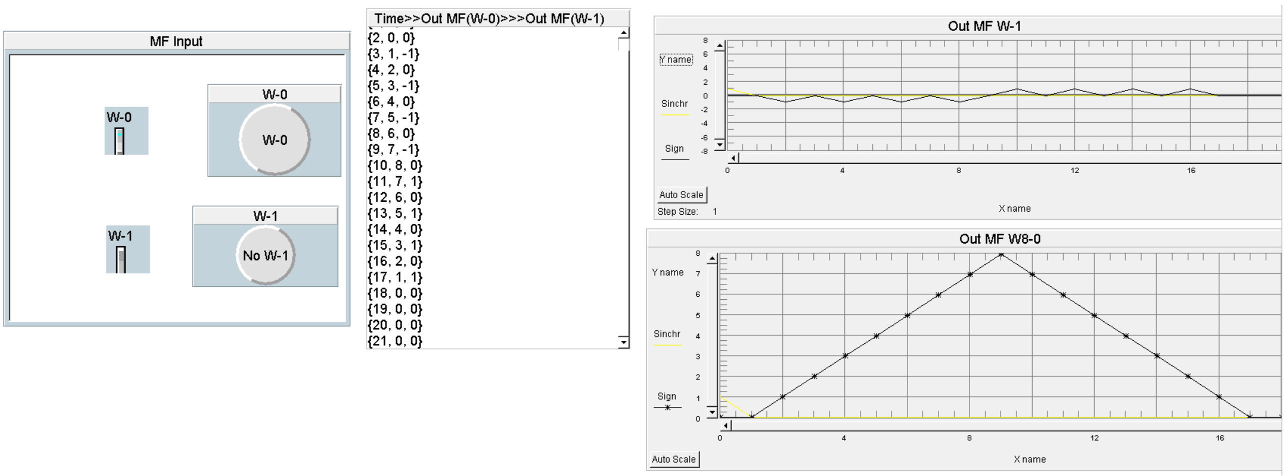


Figure 5 – Correlation functions of Walsh sequences under the action of $w_0$ at the input

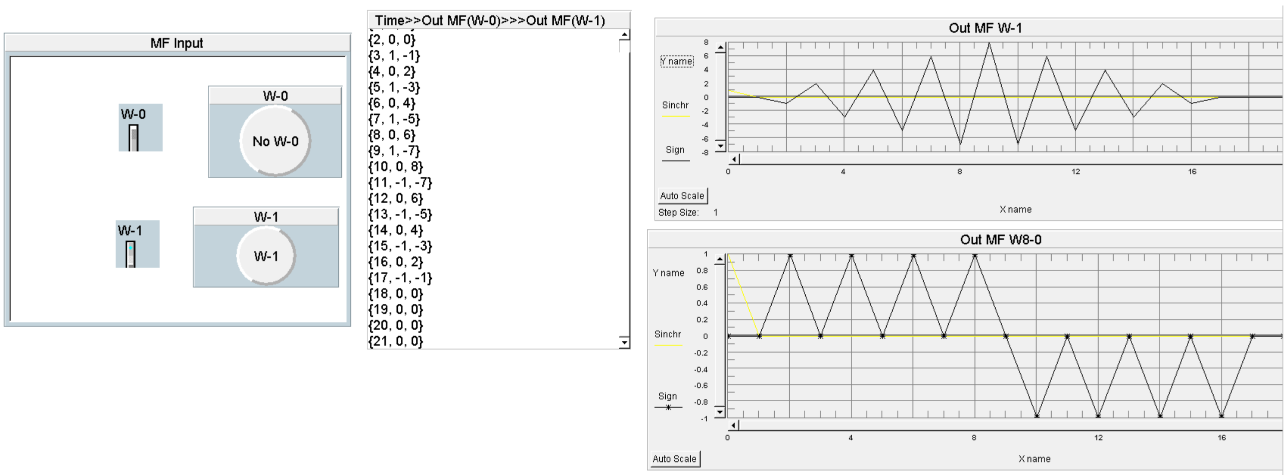


Figure 6 – Correlation functions of Walsh sequences under the action of $w_1$ at the input

The results confirm the *orthogonality* of the selected Walsh sequences. Thus, according to Fig. 5 and 6 tables, the ACF and CCF values at the reference time $t_0 = 10$ are :

$$\left[R(w_0, w_0)\right] = \left[w_0\right]\left[h(w_0)\right] = R_{\max} = m = 8, \tag{16}$$

$$\left[R(w_0, w_1)\right] = \left[w_0\right]\left[h(w_1)\right] = 0; \tag{17}$$

$$\left[R(w_1, w_1)\right] = \left[w_1\right]\left[h(w_1)\right] = R_{\max} = m = 8, \tag{18}$$

$$\left[R(w_1, w_0)\right] = \left[w_1\right]\left[h(w_0)\right] = 0. \tag{19}$$

According to (17) и (19) the values of the CCF of the selected sequences at the reference time $t_0 = 10$ are 0, and the ACF values (16) and (18) get the maximum values $R_{\max} = m = 8$, equal to the length of sequences $m = 8$.

**Evaluation of opportunities for practical application of composite sequences.** The following properties of composite sequences that are important for practical application are defined above:

1. The *property of self-synchronization* caused by the use of Barker codes as a generating sequence is useful in transmission systems for building time synchronization systems in which special synchronization sequences are used. As noted above (see (12)), with the appropriate choice of the parameters of the elementary and generating sequences, it is possible to obtain the required ACF spike of the sync sequence.
2. The *increase of noise immunity* of the ACF maximum sample due to the accumulation of symbol values in the matched filter. When selecting large values of the elementary sequence lengths *m* and generating sequence *M*, it is possible to obtain a *high signal-to-noise ratio* upon registration.
3. The *orthogonality property* due to the use of Walsh sequences as elementary sequences. The property of orthogonality in conjunction with the self-synchronization property is appropriate to use in systems with code division (CDMA), in which, according to [6, Sect. 11.4. Synchronization channel] to ensure time synchronization, long enough sync sequences are used.

The orthogonality property allows to solve the practically important increase in the specific information transfer rate carried by one carrier. Traditionally, quadrature amplitude modulation (QAM) [8] is used in such cases, for the implementation of which the *phase compaction* of the signal space is actually applied. It is important that the *separation* of quadrature channels during demodulation requires the use of *coherent* detection, which requires the *preliminary formation of a coherent oscillation.* The orthogonality property of the Walsh sequences and deriving from it the property of composite sequences orthogonality *allow the simultaneous transmission of several composite signals on the same carrier,* followed by their separation with respective *matched filters*. We will explain this with the following example.

According to the the record (8), we form composite sequences modulated in amplitude by information symbols $U_1$ $U_2$:

$$\left[U_1 C7 / w_0\right] = [U_1 w_0 c_1 \;\; U_1 w_0 c_2 \;\; U_1 w_0 c_3 \;\; U_1 w_0 c_4 \;\; U_1 w_0 c_5 \;\; U_1 w_0 c_6 \;\; U_1 w_0 c_7]; \tag{20}$$

$$\left[U_2 C7 / w_1\right] = [U_2 w_1 c_1 \;\; U_2 w_1 c_2 \;\; U_2 w_1 c_3 \;\; U_2 w_1 c_4 U_2 w_1 c_5 \;\; U_2 w_1 c_6 \;\; U_2 w_1 c_7]. \tag{21}$$

We assume that the sum of the modulated signals is being transmitted over the channel:

$$S_{chan} = \left[U_1 C7 / w_0\right] + \left[U_2 C7 / w_1\right]. \tag{22}$$

To select the transmitted information symbols, we use the matched filtering of the transferred sum (22). It follows from the form of the sum (22) that for the selection of the symbol $U_1$ it suffices to use a filter matched to the elementary sequence $w_0$. The processing of the first term (20) of this sum by a matched filter with the impulse response $h(w_0)$, taking into account the result of the ACF calculation (16), gives

$$S_1 = [U_1 C7 / w_0][h(w_0)] = U_1 m[C7]. \quad (23)$$

At the same time, the result of processing by the filter with the impulse response $h(w_1)$ taking into account the result of the ACF calculation (16) gives (due to the orthogonality property) the zero result

$$S_2 = [U_1 C7 / w_0][h(w_1)] = 0. \quad (24)$$

Here, like (23), it is possible to transmit a message symbol $U_2$

$$S_2 = [U_1 C7 / w_1][h(w_1)] = U_2 M[C7]. \quad (25)$$

From the kind of the results (23), (25) of the processing by filters matched to the elementary sequences $w_0$ and $w_1$ used, it follows that it is possible to obtain estimates of the transmitted information symbols $U_1$, $U_2$. To solve this problem, it is necessary to further process the results of (23) and (25) with a filter matched to the generating sequence *C*7, whose impulse response is determined by the expression (10). As a result of processing the SF, taking into account the properties of the MF (11), the outputs (23) and (25) increase by the generating sequence length *M* and will be equal to

$$S_1 = [U_1 C7 / w_0][h(w_0)] = U_1 m[C7][h(C7)] = mMU_1; \quad (26)$$

$$S_2 = [U_2 C7 / w_1][h(w_1)] = U_2 m[C7][h(C7)] = mMU_2. \quad (27)$$

Here, like in (24):

$$S_1 = [U_1 C7 / w_0][h(w_1)] = 0 \quad (28)$$

and

$$S_2 = [U_2 C7 / w_1][h(w_0)] = 0. \quad (29)$$

That is, calculations (28) and (29) are affected by properties of the orthogonality of elementary sequences, which determines the possibility of separating signals during demodulation. This possibility of simultaneous and independent transmission of various modulated composite sequences is verified by modeling. The working panel of the program is shown in Fig. 7. The sum of the modulated composite signals (22) is fed to the inputs of the respective matched filters, which provide *processing and signal separation*. The outputs of the MF are presented in the time diagrams. When modulating symbols $U_1 = 1$ and $U_2 = 1$ are transmitted the values of the outputs of the MF are exactly the values determined by formulas (26) and (27):

$$S_1 = mMU_1 = 56; \; S_2 = mMU_2 = 56.$$

Moreover, the shape of the ACF of each signal, despite the presence of another composite signal in the sum (22), is close to the theoretical form (see Fig. 3).

Thus, the possibility of *simultaneous* and *independent* transmission of two messages on a common channel has been theoretically and experimentally proved. It should also be noted that the number of simultaneously transmitted messages with the possibility of their subsequent separation can be *greater than two*. In other words, the *multiplicity* of such transmission (the number of simultaneously transmitted messages) is determined by the number of mutually orthogonal elementary sequences used in the composition of the composite sequences.

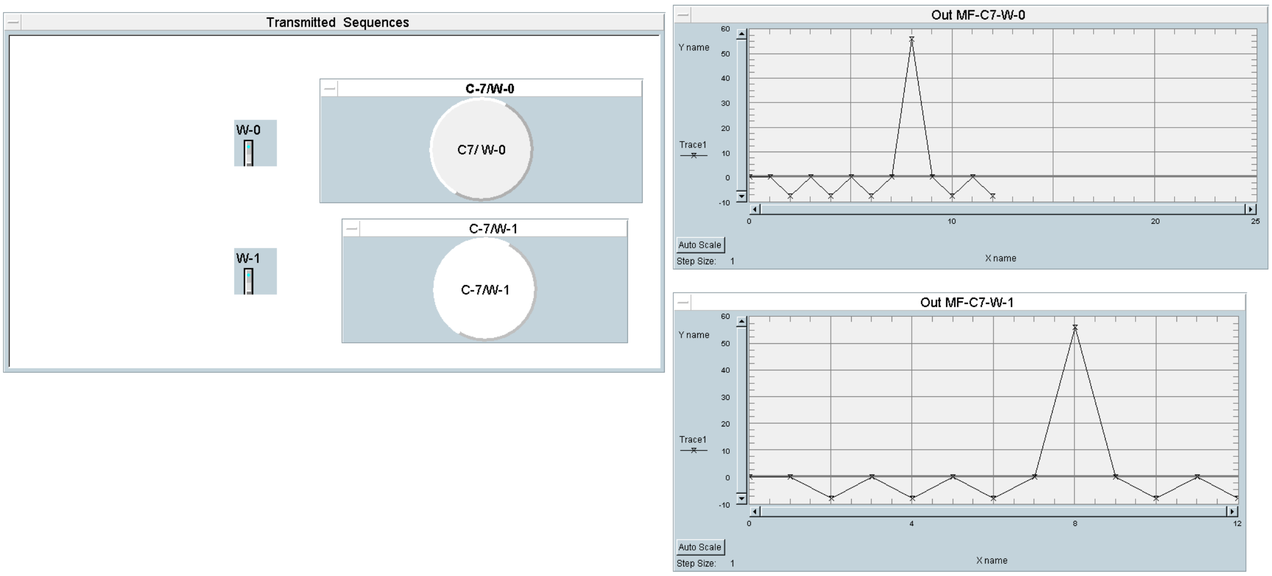


Figure 7 – The working panel of the program for verifying the possibilities of independent transmission of digital information by modulated composite sequences

It is of interest to combine the above incoherent method of simultaneous and independent transmission of several information flows with the method of narrowband continuous phase modulation (CPM). A noncoherent method for demodulating such a signal (the so-called active filter), which is stable to the effect of phase distortions in channels with quasistationary fading, was developed in [7]. The structure of such a system is shown in Fig. 8.

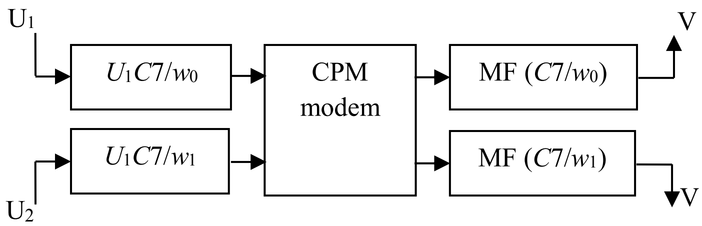


Figure 8 – Structure of the method for transmitting digital information over a channel with CPM signals

**Conclusion.** The work proposed and investigated a *new class* of so-called composite Walsh-Barker sequences. The ways of their application in perspective transmission systems are outlined.

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