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NEW DIFFERENTIAL SIGNAL-CODE CONSTRUCTIONS WITH INTERNAL CPM SIGNALS

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НОВІ ДИФЕРЕНЦІАЛЬНІ СИГНАЛЬНО-КОДОВІ КОНСТРУКЦІЇ З ВНУТРІШНІМИ СИГНАЛАМИ ЧАСТОТНОЇ МОДУЛЯЦІЇ З НЕПЕРЕРВНОЮ ФАЗОЮ

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НОВЫЕ ДИФФЕРЕНЦИАЛЬНЫЕ СИГНАЛЬНО-КОДОВЫЕ КОНСТРУКЦИИ С ВНУТРЕННИМИ СИГНАЛАМИ ЧАСТОТНОЙ МОДУЛЯЦИИ С НЕПРЕРЫВНОЙ ФАЗОЙ

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Abstract. In the article the algorithm and programs for the exhaustive search of differential signal-code constructions (DSCC) for slow fading channels are developed. The generating polynomials of optimal codes are found. A methodology for comparing the characteristics of new DSCC with the characteristics known from sources has been developed. The values of the free distance found by the DSCC significantly exceed the analogs known from the literature where a coherent demodulation method is assumed, which excludes the possibility of using the results of that work in channels with fading and phase distortion. The resulting new differential signal-code constructions may be used in quasi-stationary channels with slow fading.

Key words: differential signal-to-code design, multiposition signals, slow fading channel.

Анотація. У статті розроблено алгоритм і програми для переборного пошуку параметрів диференціальних сигнально-кодових конструкцій (ДСКК) з внутрішніми сигналами диференціальної частотної модуляції з неперервною фазою для каналів з повільними завмираннями. Знайдено породжуючі многочлени зовнішніх згорткових кодів для оптимальних сигнально-кодових конструкцій. Також розроблено методику порівняння характеристик нових ДСКК з характеристиками, відомими із зарубіжних джерел. Знайдені ДСКК за величинами вільного простору значно перевищують відомі аналоги з літератури де передбачається когерентний метод демодуляції, що виключає можливість використання результатів цієї роботи в каналах з завмираннями і фазовими спотвореннями. Отримані нові диференціальні сигнально-кодові конструкції можуть застосовуватися в квазістаціонарних каналах з повільними завмираннями.

Ключові слова: сигнально-кодові конструкції, багатопозиційні сигнали, диференціальна модуляція, канал із завмираннями.

Аннотация. В статье разработаны алгоритм и программы для переборного поиска параметров дифференциальных сигнально-кодовых конструкций (ДСКК) с внутренними сигналами дифференциальной частотной модуляции с непрерывной фазой для каналов с медленными замираниями. Найдены порождающие многочлены внешних свёрточных кодов для оптимальных сигнально-кодовых конструкций. Также разработана методика сравнения характеристик новых ДСКК с характеристиками, известными из зарубежных источников. Найденные ДСКК по величинам свободного расстояния значительно превышают известные аналоги из литературы где предполагается когерентный метод демодуляции, что исключает возможность использования результатов этой работы в каналах с замираниями и фазовыми искажениями. Полученные новые дифференциальные сигнально-кодовые конструкции могут применяться в квазистационарных каналах с медленными замираниями.

Ключевые слова: сигнально-кодовые конструкции, многопозиционные сигналы, дифференциальная модуляция, канал с замираниями.

In the work [1] theoretical results on structure synthesis and analysis of the characteristics of a new class of group differential signal-code constructions (DSCC) for channels with slow fading were presented and a theorem was proved that determines the group structure of the first difference of the modulated parameters of a pair of signal-carriers in the differential transmission system of information when using the signals of group multiposition ensembles with alphabets of symbols from additive cyclic groups with the operation of addition modulo of integers. It is shown that the consequence of this theorem is the fact that DSCC belongs to the class of invariant signal-code constructions [1] that allow not only the simple application of the test packet [1] method for searching and optimizing the generating polynomials of external convolutional codes for SCCs, but also DSCC simulation for determination of noise immunity in fading conditions and the action of additive Gaussian noise. *The task of this work* is to develop a method and algorithms of searching for new invariant DSCCs for fading channels, and to search for new DSCCs based on the developed method and algorithms.

Multiposition of CPM signals. The discrete continuous phase modulation (CPM) signal has the form [1]

$$s(t) = \sqrt{\frac{2E}{T}} \cos[\omega_c t + \varphi(t)], \tag{1}$$

where the current phase at the n-th interval $[\,nT < t \le (n+1)T\,]$ is

$$\varphi(t) = 2\pi h \sum_{i=-\infty}^{\infty} (2\alpha_i - m + 1) g_\varphi(t - iT)\,. \tag{2}$$

Here, E is the symbol energy of duration T; ω_c – signal frequency; h – modulation index; u_k – modulating symbols chosen from the alphabet of integers, $g_\varphi(t)$ – form of the phase smoothing impulse. To describe the signals, the form of the *frequency impulse* $\varphi(t)=2\pi h\sum_{i=-\infty}^{\infty}(2\alpha_i-m+1)g_\varphi(t-iT)$ is given, which is associated with the *phase impulse* $\varphi(t)=2\pi h\sum_{i=-\infty}^{\infty}(2\alpha_i-m+1)g_\varphi(t-iT)$ by the known relation $g_f(t) = \frac{d}{dt} g_\phi(t)$. Practical application finds the form of the frequency smoothing impulse of the "raised cosine" $q_f(t) = \frac{1}{2T}[1 - \cos(\frac{2\pi t}{T})], 0 \le t \le T \quad 0 \le t \le T$. It follows from expressions (1) and (2) that when considering the CPM signals as carriers of information in the DSCC, the modulated parameter should be taken to be the value of the current phase of the signal (2) taken at the corresponding time t_n. It should also be noted that the method of forming a CPM signal in the form of expressions (1) and (2) proposed by the Swedish authors [7] pursued the *maintenance of the compactness* of the energy spectrum of the signal due to the *smoothed form* of the frequency impulse $\varphi(t)=2\pi h\sum_{i=-\infty}^{\infty}(2\alpha_i-m+1)g_\varphi(t-iT)$, *simultaneously implements the differential transmission method.* As described earlier in [1], this circumstance had not been noted by the authors of the method [7] and was not used later. At CPM the transferred information is concluded in the form of continuations of phase trajectories which end with the values of phases, multiple to πh. In the main paper [1] it was noted that the CPM belongs to the class of differential modulation signals and can be used as internal signals within the DSCC.

Depending on the value of the modulation index *h*, it is possible to obtain the required value of the number of signal positions *M*. The interval for changing the index $h = \{0...hv_{\max}\}$ and the step of changing the index δh are preselected. With such parameters, the number of positions of a FM signal acquires *M* possible values of $M = \frac{h_{\max}}{\delta h}$, that allows to consider the signal (1) as the multi-position and on this basis to use such a signal to increase the specific data transfer rate. When synthesizing a SCC, the problem of matching the output of the external code codec with the input of the CPM signal modulator arises. We use an external non-binary code with an alphabet from the additive residue ring of integers modulo *M*. To match with the external code encoder, the alphabet of modulating symbols must have a group structure. We choose the set of symbols of the external code from the set of symbols of the additive algebraic group with the operation of adding integers modulo M. The alphabets are exchanged for the binary information symbols with the internal symbols of the CPM DSCC according to the rules of the Gray code. As was noted earlier, the formation of CPM signals in accordance with formulas (1), (2) provides *differential formation* of channel signals. Signal-code constructions with CPM signals belong to the class of *invariant* signal-code constructions [1, Sec. 2.11, form. (2.39)]. It was noted in the same monograph [1, Sec. 5.5].

Methodology and the algorithm of brute-force searching the optimal DSCC. In real systems, phase noise cancellation is realized by applying a differential demodulator preceding the convolutional decoder using the Viterbi algorithm.

The method of searching *optimal* codes:

1. Optimal convolutional codes whose *free distance* is *close to the upper boundary* (but does not exceed it) are found. The solution of this problem is carried out in two stages:

1.1."**Search**"stage provides a brute force search for generating polynomials.

1.2."**Verification**"stage provides verification of search results.

2. For the experimental determination of the distance of the code, the "test-packet" method is used.

3. The parameters of the investigated DSCC are preselected:

3.1. For the CPM-*M* modulation method, the modulus *M* value is being set.

3.2. Code rate *R* (the initial code rate is *R* = 1/2 in all versions of the program.) Other rates could be set using perforation.

3.3. The length of the encoding register is indicated in the program name. The choice of the corresponding program determines the length of the encoding register *K*.

4. The encoder tests are performed cyclically. Each cycle involves the formation of a test packet, its passage through a convolutional encoder, a modulator, and a path weight analysis. The program provides the setting of the test cycles number (Stop NC = 500).

5. The upper bound of the free distance of the encoder with the selected structure is determined experimentally. To that end, at the "search" stage at the beginning of each test cycle, randomly selected coefficients of the generating polynomials *G* are programmed into the encoder structure with predefined parameters (see sections 3.1…3.2 above). In this case, the current indication of the coefficients of the generating polynomials is performed.

6. If for each test cycle (i.e. for each passage of the test packet through the encoder), the output of the path weight analyzer fixes the quadratic weight W_n^2, then the average weight of the random paths is determined by averaging over the entire set of cycles of the volume NC. It was noted above that such an average weight (an average distance for invariant codes) can serve as the upper boundary of the free distance

$$D^2_{UB} = \frac{1}{N}\sum_{n=1}^{N} W_n^2 . \quad (3)$$

The upper limit value is measured at the "search" stage with a sufficiently large number of cycles NC = 500. (Switch "Search CC, UB" is in the "UB" position). After stopping the program, the result of the UB determination appears in the "Mean D2 = UB" window. This result is stored in the "StateUB" window, and by this result the search boundaries of the generating polynomial "UB" and "0,95UB" are calculated.

7. At the output of the path weight integrator, a threshold circuit is installed that fixes the result of measuring the weight W_n^2 in the interval [Δ = «UB» – «0.95UB»], that is located below from the level of the experimental upper limit, i.e. the condition (3) is satisfied, (the value of the interval Δ is in this case a small fraction of D^2_{UB} (approximately 5%)

$$(D^2_{UB} - \Delta) < W_n^2 < D^2_{UB} \quad . \tag{4}$$

8. Next, the number of cycles N is set to be large (NC = 500) and the search mode (the switch is in the "Search ConvCode" position) is started, and the code search process is started one by one with the verification of the optimal code detection condition (4). When this condition is met, the enumeration process is terminated. The value of the free distance "Result = Free Distance" is fixed, and the values of the generating polynomials are displayed on the "RandG-1" and "RandG-2" indicators and are recorded in the test report. In the monograph [1, Sec. 6.3] it is shown that such a search process can *double the average time spent* on finding the optimal code (in comparison with the time spent on searching for the full volume).

9. At the final stage of the search, the results found in step 8 are checked. To do this, in the "verification" mode instead of the randomly set values of the coefficients of the generating polynomials, fixed values of the generating polynomials, found in step 8 and entered in the protocol, are introduced. Then the search process ("Search ConvCode") is started with the simultaneous measurement of the weight W_n^2. The search stops if condition (4) is satisfied, when after setting the fixed values of the generating polynomials from the protocol, the value of the weight W_n^2 appears again within the interval Δ located near the upper boundary of the free distance. In this case, the found value of the free distance in the window "Result = Free Distance" coincides as a rule with that one previously found in step 8. In the case of such a coincidence, the values of the generating polynomials and the weight of the found code should be considered as final. In fact, the test of step 9 allows setting the form of the test packet at the encoder input, which when passing through the encoder, gives a path at the output with a weight equal to the value of the SCC free distance found.

New differential SCC with internal CPM signals. The section presents the results of the search for new differential SCCs with CPM signals and the results of comparison of the free distances of new SCCs with similar parameters known from the literature. In the monograph [7], in addition to in-depth study of the properties of CPM signals, questions of convolutional coding in the CPM channel were considered. The concept of the square of the normalized free distance is determined on the basis of the analysis of the expression for the probability of a decoding error. The mentioned square of the normalized free distance is interpreted by the authors of [7] as a certain coefficient, increasing the signal-to-noise ratio at the input of the decoder of the coherent demodulator $P_e = Q((d^2_{\min} \frac{E_b}{N_0})^{1/2})$, where the square of the normalized distance $d^2_{\min} = \min(u_\alpha, u_\beta) = \frac{1}{2E_b} \int_0^\infty [s(t,\alpha) - s(t,\beta]^2 \, dt$,

and (u_α, u_β) – compared ways of the trellis diagram of the encoded FM signal.

As you can see, the normalization is carried out according to the value of $2E_b$. In order to compare the characteristics found in this work, we can perform normalization of the free distance. Previously the free distance was determined in the process of sorting at least the weight for all possible paths on the SCC lattice diagram:

$$D^2_{(E)f} = RE_b \min[\sum_{i=1}^{T_0} (\sin \frac{2\pi h}{M} v_i)^2]. \quad (5)$$

The normalization was carried out subsequently in terms of the minimum distance of the coherent noncoded CPM. When dividing (5) into the normalizing factor $2E_b$, we obtain an expression for determining the normalized distance, which will be used for comparison with the results of the Swedish authors [7]

$$D^2_{(E)f(norm)} = \frac{R}{2} \min[\sum_{i=1}^{T0} (\sin \frac{2\pi h}{M} v_i)^2]. \quad (6)$$

The search for the minimum is made using the previously described search method.

It was shown earlier that the free distance and the specific speed of SCC increase (with an appropriate choice of external codes) with an increase in the volume of the alphabet *M*. For definiteness of the subsequent references all lines of the table are named by conditional numbers of the used code. Therefore, it is reasonable to compare the characteristics of the new DSCC with the results of the Swedish authors [7] (see Tab. 1).

Table 1 – Main characteristics of coded systems with modulation of CPM from [7]

Code number	Source [7]	Code rate *R*	Alphabet size *M*	FM index *h*	Code constraint length *v*	Specific speed γ (bits/char.)	Norm. free dist.
C-1	Table D.3 p. 486	1/2	4	0,5	1	1	3,0
C-2	Table D.6 p. 488	2/3	8	1/4	1	2	2,0

The universal program (K = 2) of the FM (NORM) determination using the formula (6) of the upper boundary of the free distance with a short (K = 2) external convolutional code in the structure of the DSCC, with various design parameters provides information transmission at different specific rates $\gamma = 1$, $\gamma = 2$ и $\gamma = 3$. The results of the program are shown in Table 2, as well as for an important case of the high specific rate of DSCC $\gamma = 3$.

The data in Table 1 allow comparison of the final characteristics of the new DSCC with the characteristics of the encoded systems of the CPM from [7]. Comparison should be carried out at the same specific speed γ. The data are compared in Table. 2. For the case of new DSCCs, the table contains the values of the upper bounds of the square of the normalized free distance. For the case of codes from the monograph of the Swedish authors there are the values of the square of the normalized distance. For the specific rate value $\gamma = 3$ codes were not searched in the monograph [7]. It can be seen that at specific rates $\gamma = 1$ and $\gamma = 2$ new DSCCs provide values of the free distance exceeding the distance values from the monograph of the Swedish authors.

Table 2 – Comparison of the characteristics of new DSCCs

Specific rate	γ = 1	γ = 2	γ = 3
New DSCCs	3,654	3,547	3,307
Monograph [7]	3,0	2,0	-

Conclusion:

1. A method and an algorithm for search of external convolutional codes in the structure of DSCC based on the use of the "test-packet" method were developed. On the basis of the well-known property of the expected value of random variables, it is suggested to use the expected value as the upper limit of the value of the free distance being searched to shorten the search time.

2. A set of programs for exhaustive search of external convolutional codes for DSCCs was developed. New differential SCCs are found.

3. A methodology for comparing the characteristics of new DSCCs with characteristics known from [7] has been developed. The values of the free distance found by DSCCs are much higher than those known from the literature.

4. In the monograph of the Swedish authors [7], studies are conducted under the assumption of the coherent demodulation method. This excludes the possibility of using the results of this work in channels with fading and phase distortions.

5. At the same time, the new differential SCCs developed in this paper are suitable and recommended for use in quasi-stationary channels with slow fading.

REFERENCES:

1. Banket V.L., Manakov S.Ju. “Struktura y kharakterystyky gruppovykh syhnal'no-kodovykh konstruktsiy s vnutrennimi sygnalamy dyfferentsialnoy modulyatsii// “Tsyfrovye tekhnologii”, № 20 (2017).
2. Banket V.L., Totmina Ju.N. “Razvitie teorii differencialnyh metodov moduljacii dlja sovremennyh cifrovyh telekommunikacionnyh sistem” // “Tsyfrovye tekhnologii.”, № 10 (2011): 43–54.
3. Petrovich N.T. “Novye sposoby osushhestvlenija fazovoj telegrafii.” // Radiotehnika № 10 (1957): 7–9.
4. Okunev Ju.B. Teorija fazoraznostnoj moduljacii / Ju.B. Okunev. – M.: Svjaz'.1979. – 216 p.
5. Gnedenko B.V. Kurs teorii verojatnostej / Gnedenko B.V. – M: Nauka, 1988, – 450 p.
6. Banket V.L., Persin A.D. “Differencial'nyj metod peredachi informacii po kanalam s zamiranijami, invariantnyj k iskazhenijam signala.” Proc. of the O.S. Popov ONAT, № 1 (2014): 29–33.
7. J.B. Anderson, T. Aulin, C.-E. Sundberg. Digital Phase Modulation. N.Y.: Plenum Press. 1986. – 490 p.
8. Banket V.L, Persin A.D. “Struktury i harakteristiki aktivnyh fil'trov dlja optimal'noj nekogerentnoj demoduljacii signalov differencial'noj FM.” «Tsyfrovye tekhnologii», № 13 (2013): 47-60.

ЛИТЕРАТУРА:

1. Банкет В.Л. Структура и характеристики групповых сигнально-кодовых конструкций с внутренними сигналами дифференциальной модуляции / В.Л. Банкет, С.Ю. Манаков // Цифровые технологии. – № 20. – 2017.
2. Банкет В.Л. Развитие теории дифференциальных методов модуляции для современных цифровых телекоммуникационных систем / В.Л. Банкет, Ю.Н Тотмина // Цифровые технологи. – 2011. – № 10. – С. 43–54.
3. Петрович Н.Т. Новые способы осуществления фазовой телеграфии // Радиотехника, 1957. – № 10. – С. 7–9.
4. Окунев Ю.Б. Теория фазоразностной модуляции / Ю.Б. Окунев. – М.: Связь. – 1979. – 216 с.
5. Гнеденко Б.В. Курс теории вероятностей / Гнеденко Б.В. – М: Наука. – 1988. – 450 с.
6. Банкет В.Л. Дифференциальный метод передачи информации по каналам с замираниями, инвариантный к искажениям сигнала / В.Л. Банкет, А.Д. Персин // Наукові праці ОНАЗ. – 2014. – № 1.– С. 29–33.
7. J.B. Anderson, T. Aulin, C.-E. Sundberg. Digital Phase Modulation. N.Y.: Plenum Press. 1986. – 490 p.
8. Банкет В.Л., Структуры и характеристики активных фильтров для оптимальной некогерентной демодуляции сигналов дифференциальной ФМ / В.Л. Банкет, А.Д. Персин // Цифрові технології. – 2013. – № 13. – С. 47–60.