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ACTIVE FILTER FOR OPTIMUM NONCOHERENT DEMODULATION OF CONTINUOUS PHASE FREQUENCY MODULATION SIGNALS

Banket V.L., Manakov S.Ju.

O.S. Popov Odessa national academy of telecommunications
Kuznechna st., 1, Odessa, Ukraine, 65029
vlbank@mail.ru, manakov@onat.edu.ua

АКТИВНИЙ ФІЛЬТР ДЛЯ ОПТИМАЛЬНОЇ НЕКОГЕРЕНТНОЇ ДЕМОДУЛЯЦІЇ СИГНАЛІВ ЧАСТОТНОЇ МОДУЛЯЦІЇ З НЕПЕРЕРВНОЮ ФАЗОЮ

Банкет В.Л., Манаков С.Ю.

Одеська національна академія зв'язку імені О.С.Попова,
вул. Кузнечна, 1, Одеса, Україна, 65029
vlbank@mail.ru, manakov@onat.edu.ua

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Банкет В.Л., Манаков С.Ю.

Одесская национальная академия связи имени А.С. Попова,
ул. Кузнечная, 1, Одесса, Украина, 65029
vlbank@mail.ru, manakov@onat.edu.ua

Abstract. In article the algorithm and structure of the active filter-demodulator for continuous phase frequency modulation signals are developed.

Key words: active filter-demodulator, continuous phase frequency modulation signals.

Анотація. В статті розроблені алгоритм та структура активного фільтра-демодулятора сигналів частотної модуляції з неперервною фазою.

Ключові слова: активний фільтр-демодулятор, сигнал частотної модуляції з неперервною фазою.

Аннотация. В статье разработан алгоритм и структура активного фильтра-демодулятора сигналов частотной модуляции з непрерывной фазою.

Ключевые слова: активный фильтр-демодулятор, сигнал частотной модуляции с непрерывной фазой.

Introduction. Formulation of the problem. Discrete signals of continuous phase frequency modulation (CPFM) are characterized by a constant envelope, absence of phase jumps, and high indicators of power and frequency efficiency. All these advantages in a combination with compactness of a spectrum and low level of out-of-band emission have determined wide application of CPFM signals in systems of ground and satellite communications. Methods of forming and modulation/demodulation of CPFM signals are detailed in the fundamental monograph [1], and in the popular guide [3]. A detailed analysis of non-coherent processing methods is performed by a group of authors [2] under the guidance of dr. L.H. Lampe. Monographs [4, 5] are devoted to the consideration of the same issues. For estimation of capabilities of practical appliance of CPFM a demodulating algorithm is important. In [1] a significant attention is paid to designing an optimal (by the criterion of maximum likelihood) coherent CPFM demodulation algorithm [1, section 7.1.1], in accordance to which the demodulator includes a set (bank) of possible signal realizations and the decision is made by results of correlative processing of the received sum of signal with noise on a certain time interval. At the same time, authors of this monograph, noting complexity of realization of optimum coherent reception, consider possibilities of non-coherent reception of CPFM [1, section 7.1.2]. An autocorrelation algorithm is used as the basic non-coherent algorithm. The additional analysis have shown that CPFM signal belongs to the class of *differential modulation* signals, and that on to this basis it is possible a development of *algorithm* based on idea of creation of the so-called “active” filter [6] for demodulation of such signal. *Article task* is the development of algorithm and structures of the active filter for non-coherent demodulation of CPFM signals.

CPFM as a signal of the differential modulation. Discrete CPFM signal looks like [1]

$$s(t)=\sqrt{\frac{2E}{T}}\cos\left[\omega_c t+\phi(t)\right], \tag{1}$$

where the current phase on *n*-th interval $[\,nT<t\le(n+1)T]$ is equal to

$$\phi_{(n)}(t)=2\pi h\sum_{k\le n}u_k g_\phi(t-kT). \tag{2}$$

Here E – energy of a symbol with duration of T, ω_c – frequency of a signal, h – index of modulation,

u_k – modulating symbols, chosen from the alphabet: $u_k\in\left[\pm1,\pm2,\ldots\pm3,\ldots\pm(M-1)\right]$,

$g_\phi(t)$ – a form of a phase smoothing impulse. By definition [1] as function of a phase smoothing impulse $g_\phi(t)$ any function, satisfying to following conditions can be used:

1) At $t\le0$ function $g_\phi(t)=0$;

2) At $t\ge T$ function $g_\phi(t)=\frac{1}{2}$, where T – an interval of time. (3)

The details about typical forms of phase functions are given in the monograph [4, tab. 2.3]. The most widespread example, which is used for the analysis, is the form of a phase impulse given below, called in literature as REC (a rectangular shape of a frequency impulse to which there corresponds a linear function of phase change)

$$g_\phi(t)=\begin{cases}0 & , \quad t<0,\\ \frac{t}{2T} & , \quad 0\le t\le T,\\ \frac{1}{2} & , \quad t>T.\end{cases} \tag{4}$$

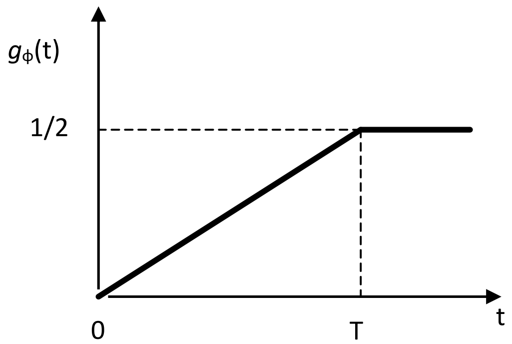

Figure 1 – Phase function of a CPFM signal (REC)

CPFM signal is a signal “with memory” [2]. Indeed, according to a formula for the current phase (2) in the form of the phase trajectory it is stored all the information about the long process of phase formation and new phase increment values are determined by values and signs of the transferred information symbols. The shapes of phase trajectories in the form of a so-called «phase tree» are presented on Fig. 2. Thin lines represent possible phase trajectories. It is visible that ruptures of phase trajectories are absent. Trajectories meet in certain points (knots). Bold lines note the phase trajectory, corresponding to transfer of the information symbols sequence *u* = +1, – 1, – 1, +1, +1, ... I.e., the positive inclination of a piece of the phase trajectory corresponds to transfer of the positive symbol, and the negative inclination – to transfer of the negative symbol. All possible trajectories pass through nodal points located at levels, multiple to size *πh*.

Thus, in process of transferring in the modulator according to a formula (2) the sequence of phases of FM signals is formed

$$...\phi_{(n-2)}(t), \phi_{(n-1)}(t), \phi_{(n)}(t)... \tag{5}$$

Then the sequence of signals (1) is sent to the channel with such phases

$$...s\left[t, \phi_{(n-2)}(t)\right], s\left[t, \phi_{(n-1)}(t)\right], s\left[t, \phi_{(n)}(t)\right].... \tag{5a}$$

It is known that at a differential type of modulation information is transferred by changes of parameters of two next signals in time. In this case it is the signals forming a *pair* $\left\{s\left[t, \phi_{(n-1)}(t)\right], s\left[t, \phi_{(n)}(t)\right]\right\}$. Differently, *information is put in changes of the phase parameters* $\left\{\phi_{(n-1)}(t), \phi_{(n)}(t)\right\}$ of the signals in this pair. Let's consider the representation of the signals by duration of *N* symbols which are set on the corresponding time intervals:

1) Signal on “previous” (n–1)-th time interval $\left[(n-1-N)T \le t < (n-1)T\right]$

$$s_{(n-1)}(t) = \sqrt{\frac{2E}{T}} \cos\left[\omega_c t + \phi_{(n-1)}(t)\right] \tag{6}$$

with a phase

$$\phi_{(n-1)}(t) = 2\pi h \sum_{k \le n-1} u_k g_\phi (t - kT);$$

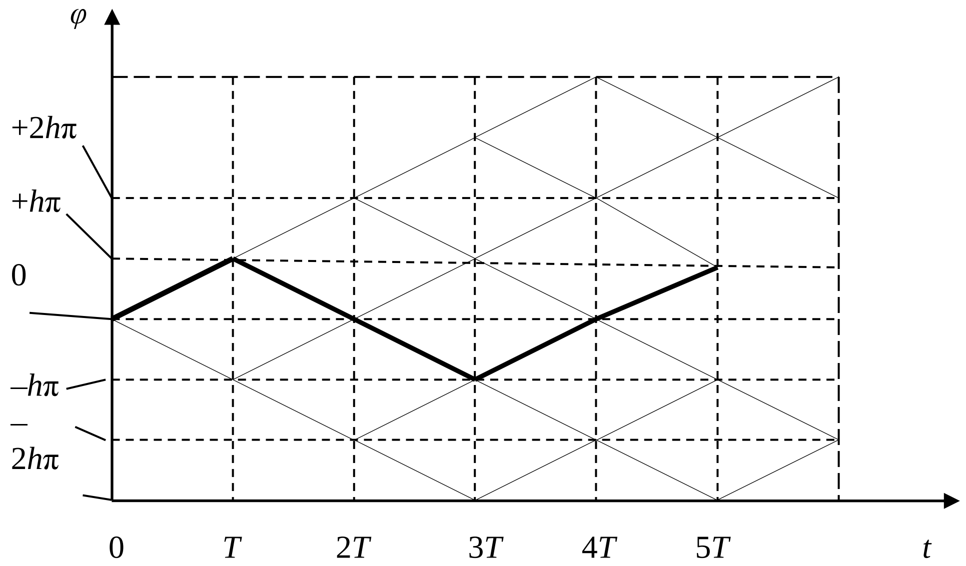

Figure 2 – Phase trajectories of CPFM (REC) signal

2) Signal on “the following” n-th time interval $\left[(n-N)T \le t < nT\right]$

$$s_{(n)}(t) = \sqrt{\frac{2E}{T}} \cos\left[\omega_c t + \phi_{(n)}(t)\right] \tag{7}$$

with a phase

$$\phi_{(n)}(t) = 2\pi h \sum_{k \le n} u_k g_\phi (t - kT).$$

Both signals ("the previous" $s[t,\phi_{(n-1)}(t)]$ and "the following" $s\left[t,\phi_{(n)}(t)\right]$) at such definition are placed within same time interval $\left\{(n-1-N)T \le t < nT\right\}$.

Owing to specific features of phase impulses forms (3), phase trajectories of the signals (6) and (7) end in the values, multiple to size *πh*. It is easy to be convinced of it, having substituted the argument values in the formulas of phase impulses, corresponding to the end of signal: at $(t = nT)$ $g_\phi(nT) = \frac{1}{2}$ for the large values of n.

Thus, in CPFM the transmitted information is contained in the form of extensions of the phase trajectories, which come to an end with the values of phases, multiple to *πh*. Calculation of the first difference of current phases has the interest for the subsequent:

$$\Delta^1_{(n)}\left(\phi_{(n)}(t)\right) = \phi_{(n)}(t) - \phi_{(n-1)}(t) = 2\pi h\left\{\sum_{k\le n} u_k g(t-kT) - \sum_{k\le(n-1)} u_k g(t-kT)\right\} = 2\pi h u_n g(t-kT) = \pi h u_{(n)}.$$

From here follows the communication of estimates of transferred information symbols with estimates of the first differences of phases, subsequently designated as $\Delta^1_{(n)} = \Delta^1_{(n)}\left(\phi_{(n)}(t)\right)$:

$$u_{(n)} = \frac{\Delta^1_{(n)}\left(\phi_{(n)}(t)\right)}{\pi h}. \tag{8}$$

It is possible to make *direct analogy* of considered CPFM modulation method with a rule of differential phase modulation (PM). It is known that at differential PM a phase of a transferred signal $\phi_{(n)}$ is connected with a phase of the previous signal $\phi_{(n-1)}$ by the known *rule of differential coding* ($\Delta\phi$ – FM index)

$$\phi_{(n)} = \phi_{(n-1)} + \Delta\phi u_{(n)}. \tag{9}$$

In the case of CPFM on the basis of finding of the first difference of phases we receive *the rule of differential CPFM*:

$$\phi_{(n)}(t) = \phi_{(n-1)}(t) + \Delta^1_{(n)}(t). \tag{10}$$

The difference between compared modulations methods (the differential PM and CPFM) consists in the following:

1) For differential PM signals with phases $\left\{\phi_{(n-1)}, \phi_{(n)}\right\}$ *ac* are transferred consequently in time and they *are do not overlap* on a modulator exit, whereas for CPFM (having, as stated above, the *differential properties*) "previous" $s\left[t,\phi_{(n-1)}(t)\right]$ and "subsequent" $s\left[t,\phi_{(n)}(t)\right]$ signals on modulator exit *are overlapped* (*the subsequent signal is continuation of the previous*);

2) The rule of differential modulation (9) for PM is written *for discrete symbols*, whereas in case of CPFM this rule is written for *time functions (the phase trajectories)*;

3) The difference of temporal position of compared signals, as noted in 1), is necessary for considering when developing demodulation algorithm of CPFM signals: in the CPFM demodulator *it should be realized processing of signals partially being overlapped in time.*

Let us present signals (6), (7) in two-dimensional Hilbert space with orthogonal basis functions on a time interval $\left\{(n-1-N)T \le t < nT\right\}$ (covering time intervals signals (6) and (7)) and, respectively, with ort vectors $\left\{\vec{f}_1, \vec{f}^*\right\}_1$:

$$f_1(t) = \sin\omega_0 t \Rightarrow \vec{f}_1,\ f^*{}_1(t) = \cos\omega_0 t \Rightarrow \vec{f}^*_1. \tag{11}$$

Here and further "*" is a Hilbert pairing sign.

In this *auxiliary* space signals (6) and (7) are presented by vectors

$$\vec{s}_{(n-1)} \Rightarrow s_{(n-1)}(t),\ \vec{s}_{(n)} \Rightarrow s_{(n)}(t). \tag{12}$$

The spatial angle $\Delta\phi_{(n)}$ between vectors $\vec{s}_{(n-1)}$ and $\vec{s}_{(n)}$, displaying change in a phase difference between vectors in the modulation process can be calculated by known rules of vector space, using the scalar product (in denominator – the product of the norms of vectors (12))

$$\cos\Delta\phi_{(n)} = \frac{\left(\vec{s}_{(n)}\vec{s}_{(n-1)}\right)}{\left\|\vec{s}_{(n)}\right\|\left\|\vec{s}_{(n-1)}\right\|},\ \sin\Delta\phi_{(n)} = \frac{\left(\vec{s}^{\,*}_{(n)}\vec{s}_{(n-1)}\right)}{\left\|\vec{s}_{(n)}\right\|\left\|\vec{s}_{(n-1)}\right\|}\ . \tag{13}$$

Let us define projections of vectors of received signals (12) on basis ort vectors (11)

$$X_{(n)} = \int_{(n-1-N)T}^{nT} s_{(n)}(t)\sin\omega_0 t\,dt,\ X_{(n-1)} = \int_{(n-1-N)T}^{nT} s_{(n-1)}(t)\sin\omega_0 t\,dt, \tag{14}$$

$$Y_{(n)} = \int_{(n-1-N)T}^{nT} s_{(n)}(t)\cos\omega_0 t\,dt,\ Y_{(n-1)} = \int_{(n-1-N)T}^{nT} s_{(n-1)}(t)\cos\omega_0 t\,dt.$$

Geometric representation of calculation of vectors projections (14) of FM signals (12) on basic axes (11) is shown on Fig. 3. It is also marked the spatial angle $\Delta\phi_{(n)}$ between vectors.

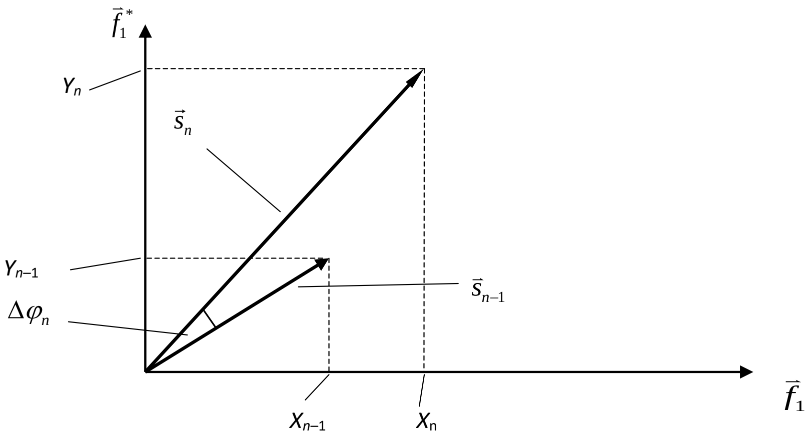

Figure 3 – A geometry of calculation of signals (12) projections

The link (8) between the estimates of the transferred symbols and the estimates of the first differences $\Delta^1_{(n)}$ allows formulating the *problem of CPFM signals demodulation* as follows. Through the channel with CPFM signals it is transferred the sequence of information symbols

$$\bar{u} = \ldots u_0 \ldots u_i \ldots\ . \tag{15}$$

Each current information symbol u_i according to the rule (2) modulates the current phases of an FM signal so, that signals are sent to the channel (1), which phases are forming the sequence of the current phases. In *the quasistationary* channel to each phase transferred, a phase shift ϕ_k is added so that the received pair of phase estimates, taking into account this shift, has the form

$$\left\{\widehat{\phi}_{(n-1)}(t), \widehat{\phi}_{(n))}(t)\right\} = \left\{\left[\phi_{(n-1)}(t)+\phi_k\right], \left[\phi_{(n)}(t)+\phi_k\right]\right\}. \tag{16}$$

In the calculation of the first phase difference in pair (16) *the phase interference is suppressed* and in the final result of calculations it is absent:

$$\Delta^1_{(n)}\left(\phi_{(n)}(t)\right) = \widehat{\phi}_{(n)}(t) - \widehat{\phi}_{(n-1)}(t) = \pi h u_{(n)}. \tag{17}$$

This property of CPFM signals, as *signals of differential modulations* can be it is used for information transfer by CPFM method through quasistationary channels with slow phase changes, brought by the channels. It is necessary to find an *algorithm for optimal non-coherent* demodulation, implementing the judgment on the transmitted symbols u_i by *maximum likelihood criterion*.

CPFM demodulation algorithm. The above-mentioned analogy between CPFM and the differential modulation method of PM signals can be used for the synthesis of a demodulation algorithm according to the procedure described in detail in [6]. It is considered the transfer of messages u_i using CPFM. To each transferred message there corresponds a pair of phases $\left\{\phi_{(n-1)}, \phi_{(n)}\right\}$, and further – a pair of transferred signals $\left\{s\left[t, \phi_{(n-1)}(t)\right], s\left[t, \phi_{(n)}(t)\right]\right\}$. The transmitted information is contained in the sequence of the first phase differences:

$$\ldots\Delta^1_{(n-2)}, \ldots, \Delta^1_{(n-1)}, \Delta^1_{(n)} \ldots. \tag{18}$$

In the channel on the transferred signal $s(t)$ the additive $n(t)$ is imposed so, that on an entrance of the demodulator the sum of both signal and noise arrives, operating on the same interval $\left[(n-1-N)T \le t < nT\right]$:

$$r(t) = s(t) + n(t). \tag{19}$$

In vector representation to the received sum (19) of the signal with the noise there corresponds a vector $\vec{r}$: $\left[r(t) = s(t) + n(t)\right] \Rightarrow \vec{r}$.

To each vector $\vec{r}$ there correspond projections to coordinate axes: $\vec{r} \Rightarrow [X_{(n-1)}, X_{(n)}, Y_{(n-1)}, Y_{(n)}]$.

Unlike the coherent receiving, in the non-coherent demodulator the processing of the received FM signal with the noise is made in auxiliary coordinate system, produced by the orthogonal basis oscillations (12), which frequencies and initial phases *are not connected in any way* with similar parameters of the transferred signals. Demodulation is performed according to the following stages:

1) The demodulator provides the possibility of estimating the current phase of the received signal *r*(*t*) with respect to said reference oscillation (12) with arbitrary phases

$$\ldots\phi_{r(n-2)}, \phi_{r(n-1)}, \phi_{r(n)}, \ldots. \tag{20}$$

For this purpose, in the demodulator there is included the calculation unit of the received signal projections to the coordinate axis (12). In the process of transmitting a pair of signals $\left\{s\left[t, \phi_{(n-1)}(t)\right], s\left[t, \phi_{(n)}(t)\right]\right\}$ is formed in the modulator and is transmitted to the channel so that each “previous” signal $s\left[t, \phi_{(n-1)}(t)\right]$ is applied to the input of the demodulator before the “following” signal $s\left[t, \phi_{(n)}(t)\right]$, forming a sequence of the form (5a). The demodulator is provided with a system clock, separating the received signal $r(t) = s(t) + n(t)$ for a continuous code of length *T* so that the input of the demodulator is affected by the pair

$$\left\{r_{(n-1)}(t), r_{(n)}(t)\right\} = \left\{\left[r_{(n-1)} = s\left(t, \phi_{(n-1)}(t)\right) + n_{(n-1)}\right], r_{\ (n)} = \left[s\left(t, \phi_{(n)}(t)\right) = n_{(n)}(t)\right]\right\}. \tag{21}$$

The calculation of the projections of the form (14) for each of the above-mentioned pair of signals is performed by the formulas:

$$X_{(n)} = \int\limits_{(n-1-N)T}^{nT} r_{(n)}(t) \sin\omega_0 t\, dt, \; X_{(n-1)} = \int\limits_{(n-1-N)T}^{nT} r_{(n-1)}(t) \sin\omega_0 t\, dt, \tag{22}$$

$$Y_{(n)} = \int\limits_{(n-1-N)T}^{nT} r_{(n)}(t) \cos\omega_0 t\, dt, \; Y_{(n-1)} = \int\limits_{(n-1-N)T}^{nT} r_{(n-1)}(t) \cos\omega_0 t\, dt.$$

It is seen that the calculation of the projections of each implementation of a couple is performed on the *same time interval* $\{(n-1-N)T \le t < nT\}$. It appears convenient for construction of projections calculation block. If the processed signal arrives from the channel in the form of sequence

$$\ldots r_{(n-2)}, r_{(n-1)}, r_{(n)} \ldots, \tag{23}$$

then calculation of every projection (*X* and *Y*) is performed by *one and the same* integrators, implementing the «*sliding*» integration within temporary "window" $\left[(n-1-N)T \le t < nT\right]$. It is possible to perform the subsequent calculation of phase differences using scalar products by the formulas, similar to (14):

$$\cos\Delta^1{}_n\phi_r = \frac{\left(\vec{r}_{(n)}\vec{r}_{(n-1)}\right)}{\left\|\vec{r}_{(n)}\right\|\left\|\vec{r}_{(n-1)}\right\|}, \quad \sin\Delta^1{}_n\phi_r = \frac{\left(\vec{r}^{\,*}_{(n)}\vec{r}_{(n-1)}\right)}{\left\|\vec{r}_{(n)}\right\|\left\|\vec{r}_{(n-1)}\right\|}\ . \qquad (24)$$

In [7] it is shown that the projections $\left[X_{(n-1)}, X_{(n)}, Y_{(n-1)}, Y_{(n)}\right]$ may define the scalar products as follows:

$$\left(\vec{r}_{(n)}\vec{r}_{(n-1)}\right) = X_{(n)(r)}X_{(n-1)((r)} + Y_{(n)}Y_{(n-1)(r)}\,,\ \left(\vec{r}^{\,*}_{(n)}\vec{r}_{(n-1)}\right) = X_{(n-1)(r)}Y_{(n)(r)} - X_{(n)(r)}Y_{(n-1)(r)}\,. \qquad (25)$$

These values of scalar products may be used in formulas (24).

2) For the subsequent extraction of estimates of the transferred information symbols the demodulator calculates a first difference between the elements of the phase sequence (5)

$$\ldots\Delta^1_{(r)(n-2)}, \Delta^1_{(r)(n-1)}, \Delta^1_{(r)(n)}\ldots. \qquad (26)$$

3) The calculation of trigonometric functions from the first phase differences $\left\{\sin\Delta^1\phi_r, \cos\Delta^1\phi_r\right\}$ is performed.

4) The task of the demodulation formulated as follows: to find an algorithm by which each pair of $\left\{\sin\Delta^1\phi_r, \cos\Delta^1\phi_r\right\}$ will be put in one correspondence with phase difference of the set of all possible variants of the phase differences of the form (18).

In other words, at first received phase difference $\Delta^1\phi_r$ it should be determined, which phase difference of the potential set (18) has been transferred. As it is known, in the statistical theory of signals discrimination for implementation of the algorithm of the optimal distinguish it is recommended the use of *decisions by the maximum a posteriori probability*. Let for all hypotheses about transfer of differences $\Delta^1\phi_i$ from a set (18) posterior probabilities $P\left(\Delta^1\phi_i / \Delta^1\phi_r\right), (i = 1\ \ldots M)$ are known. Considering hypotheses about transfer of differences $\Delta^1\phi_i$ and $\Delta^1\phi_j$, it is necessary to take out *decision* about difference $\Delta^1\phi_i$ transfer at performance of inequality

$$P\left(\Delta^1\phi_i / \Delta^1\phi_r\right) > P\left(\Delta^1\phi_j / \Delta^1\phi_r\right) \text{ (for all } i \ne j\text{)}, \qquad (27)$$

or, at the performance of such inequality expressed through likehood functions (conditional densities of probability)

$$W\left(\Delta^1\phi_x / \Delta^1\phi_i\right) > W\left(\Delta^1\phi_x / \Delta^1\phi_j\right),$$

or, equivalently, by the probability densities of differences

$$W\left(\Delta^1\phi_x - \Delta^1\phi_i\right) > W\left(\Delta^1\phi_x - \Delta^1\phi_j\right)\ . \qquad (28)$$

If functions $W\left(\Delta^1\varphi_x - \Delta^1\phi_i\right), (i = 1 \ldots M)$ – are even and monotonously decreasing functions relative to points $\left(\Delta^1\phi_x - \Delta^1\phi_i\right) = 0$, then it is possible to replace the inequality (28) with a rule in which instead of probability density any even and monotonously decreasing function, for example, the cosine function, is used. Considering it we receive the following rule of choice of the transferred phases difference $\Delta^1\phi_i$

$$\cos\left(\Delta^1\phi_r - \Delta^1\phi_i\right) > \cos\left(\Delta^1\phi_r - \Delta^1\phi_j\right)(i \ne j)\,(i \ne j\,). \qquad (29)$$

Expanding the left and right side of this inequality by the rules of trigonometry, we get

$$\cos\Delta^1\phi_r \cos\Delta^1\phi_i + \sin\Delta^1\phi_r \sin\Delta^1\phi_i > \cos\Delta^1\phi_r \cos\Delta^1\phi_j + \sin\Delta^1\varphi_r \sin\Delta^1\phi_j \quad (30)$$

Earlier expressions were received (24) for trigonometrical functions of differences of angles $\left[\cos\Delta^1\phi_r, \sin\Delta^1\phi_r\right]$ and expressions (25) for scalar products entering into them through projections $\left[X_{(n-1)}, X_{(n)}, Y_{(n-1)}, Y_{(n)}\right]$. Focusing on the demodulation of CPFM signal with the use of active filter with issuing so called "Flexible" solutions at the demodulator output (for subsequent use of optimal processing of flexible solutions, such as decisions averaging by the procedure of accumulation and use of outer convolutional coding with Viterbi algorithm) let us formulate a discrete PM *demodulation algorithm with flexible output*. Let us compare the left and right sides of the inequalities (29) and (30). Signs of conformity marked with arrows

$$P\left(\Delta\phi_j / \Delta_1\phi_x\right) \rightarrow \left[\cos\Delta_1\phi_x \cos\Delta\phi_j + \sin\Delta_1\phi_x \sin\Delta\phi_j\right]. \quad (31)$$

It is seen that values on the right in (33), proportional to posterior probabilities of differences $\Delta\phi_i$ and $\Delta\phi_j$, are formed by similar rules which can be accepted, as *algorithm of optimum CPFM demodulation with the flexible decision,* i.e. as an output of the demodulator we will consider

$$V_{FD}\left(\phi_x, \Delta\phi_i\right) = \cos\Delta_1\phi_x \cos\Delta\phi_i + \sin\Delta_1\phi_x \sin\Delta\phi_i \;. \quad (32)$$

Substituting in this expression values for the trigonometric functions of the differences (24) and then the scalar products of (25) we receive an algorithm convenient for realization:

$$V_{FD}\left(\phi_x, \phi_i\right) = C\left\{\left(X_n X_{n-1} + Y_n Y_{n-1}\right)\cos\Delta\phi_i + \left(X_{n-1} Y_n - X_n Y_{n-1}\right)\sin\Delta\phi_i\right\}. \quad (33)$$

Here constant $C = \dfrac{1}{\left\|\vec{X}_n\right\|\left\|\vec{X}_{n-1}\right\|}$. Size C does not depend on hypothesis $\Delta\phi_i$ number, on which the posterior probability is formed. From this expression follows the structure of the optimal coherent demodulation of FM signals (Fig. 4). Basis of the demodulator is the generator of orthogonal reference oscillations (OSCref) with frequency $\omega_0 \approx \omega_c$, rather close to frequency of a signal (ω_c). The divergence between the frequencies $\Delta\omega_0 = \omega_c - \omega_0$ should *vary slightly* over the duration of the transmission of signals pair $\left\{s\left[t, \phi_{(n-1)}(t)\right], s\left[t, \phi_{(n)}(t)\right]\right\}$. In this case, the phase rotation of the reference oscillations (11) by an angle $\Delta\phi_0 = T\,\Delta\omega_0$ can be related to the values of the phase shifts added by the channel, which are effectively suppressed by first demodulator when calculating first phase differences. The received sum of a signal with an interference $r(t) = s(t) + n(t)$ moves on the blocks of projections formation P_x and P_y, in which on formulas (22) it is performed the calculation of projections for (X, Y) channels. Operation of time integrators in each of X and Y channels is defined by a *tact synchronization system* (STS), and the values of projections $\left[X_{(n-1)}, X_{(n)}\right]$ and $\left[Y_{(n-1)}, Y_{(n)}\right]$ appear on an output of each calculator sequentially, and for their combination in time for calculation on the algorithm (33) there used the delay lines by tact *T*.

It is easy to receive the optimum demodulation algorithm of binary CPFM signals with the hard decision from the general algorithm (33). When transmitting information symbols $u_n = 0$ and $u_n = 1$ with modulation index *h* = 1, the alphabet of possible phase increments of a signal (1) will be $\Phi = \left\{\Delta\phi_0 = 0, ..., \Delta\phi_1 = \pi\right\}$. Thus all possible values of an output of the modulator with the flexible decision (34) will be located within range $\{0 ... \pi\}$.

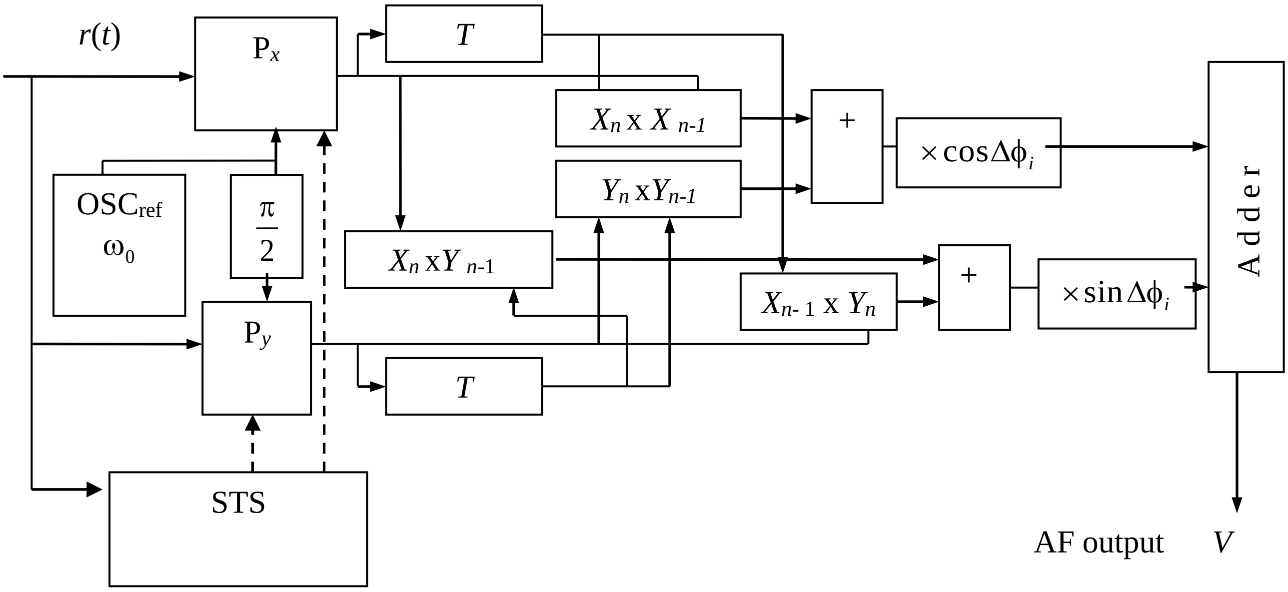

Figure 4 – The structure of the active filter for CPFM signals

For making hard decisions we divide all the possible interval using the border $\Delta\phi = \frac{\pi}{2}$ on two sub-intervals $\left\{0,...\frac{\pi}{2}\right\}$ and $\left\{\frac{\pi}{2},...\pi\right\}$. It will allow to form hard decisions on rules:

$$\text{At } \left\{0 < \left[V_{FD}\left(\Delta\varphi_x, \Delta\varphi_i = \frac{\pi}{2}\right)\right] \le \frac{\pi}{2}\right\} \text{ decision } \Delta\phi_0 = 0 \text{ and } u_n = 0,$$

$$\text{At } \left\{\frac{\pi}{2} \le \left[V_{FD}\left(\Delta\varphi_x, \Delta\varphi_i = \frac{\pi}{2}\right)\right] < \pi\right\} \text{ decision } \Delta\phi_0 = \pi \text{ and } u_n = 1. \quad (34)$$

The same result can be obtained from the algorithm (33) using a function sign(x). We substitute the values of the trigonometric functions $\cos\frac{\pi}{2} = 1$ and $\sin\frac{\pi}{2} = 0$ in the algorithm (33). The result – a hard decision algorithm – will have the form

$$V_{HD}(\phi_r, \phi_i) = \text{sign}\left\{C\left(X_n X_{n-1} + Y_n Y_{n-1}\right)\right\}. \quad (35)$$

An algorithm for extraction of the incoherent timing signal. In the noncoherent CPFM demodulator (Fig. 4) an algorithm for extraction of timing signal *should also be incoherent*. The simplest noncoherent algorithm for extraction of first phase differences is the *autocorrelation algorithm*, where on the interval of the processed signal it is being integrated a product of the received signal and its copy delayed by the symbol duration T. In the case under consideration, when the value of the signal symbol duration *T is known previously* (but *the beginning and end of the character are unknown*, i.e. integration limits), for the solution of an objective it is enough to be limited to multiplication operations. Let us consider, using known formulas of trigonometry, product of the signal of the form (1) and its delayed copies:

$$s(t) = \sqrt{\frac{2E}{T}}\cos\left[\omega_c t + \phi(t)\right], \quad \phi_{(n)}(t) = 2\pi h \sum_{k \le n} u_k g_\phi(t - kT),$$

$$s(t-T) = \sqrt{\frac{2E}{T}}\cos\left[\omega_c(t-T) + \phi(t-T)\right], \quad \phi_{(n)}(t) = 2\pi h \sum_{k \le n} u_k g_\phi(t - kT),$$

$$s(t)s(t-T)=\frac{E}{T}\left\{\cos\left[2\omega_c t-\omega_c T+\phi(t)+\phi(t-T)\right]+\cos\left[-\omega_c T+[\phi(t)-\phi(t-T)\right]\right\}. \qquad (36)$$

Further it is convenient to pass the delayed signal through the phase shifter by a corner $\frac{\pi}{2}$ (the signal is noted by a sign (*):

$$s(t)=\sqrt{\frac{2E}{T}}\cos\left[\omega_c t+\phi(t)\right], \qquad \phi_{(n)}(t)=2\pi h\sum_{k\le n}u_k g_\phi(t-kT),$$

$$s^*(t-T)=\sqrt{\frac{2E}{T}}\sin\left[\omega_c(t-T)+\phi(t-T)\right], \qquad \phi_{(n)}(t)=2\pi h\sum_{k\le n}u_k g_\phi(t-kT).$$

As a result of multiplication we receive

$$s(t)s^*(t-T)=\frac{E}{T}\left\{\cos\left[2\omega_c t-\omega_c T+\phi(t)+\phi(t-T)\right]+\cos\left[-\omega_c T+\phi(t)-\phi(t-T)\right]\right\}. \qquad (37)$$

It can be seen that the outputs of the multipliers (36) and (37) comprise a phase difference, i.e. the first difference of phases $\Delta^1(T)=\left[\phi(t)-\phi(t-T)\right]=2\pi h u_n g(t-nT)$. Summands in the curly brackets in (36), (37) with a doubled frequency $2\omega_c$ of a signal can be suppressed by low-pass filters (LPF). Taking this into account there remain sine and cosine of the same argument in expressions (36) and (37). Their multiplication gives

$$U_{ss}(n)=\left[s(t)s(t-T)\right]\left[s(t)s^*(t-T)\right]=\frac{1}{2}\sin\left\{2\left[-\omega_c T+2\pi h u_n g(t-nT)\right]\right\}. \qquad (38)$$

Thus, the above procedure of noncoherent "*pseudo correlative*" processing allows to extract a sequence of phase pulses, modulated by information symbols, from the received signal. I.e. signal (38) as the output of sync pulses extractor comprises information about the frequency and phase of that pulses. The presence of the summand $(-\omega_c T)$ in the argument is not a an interference, as the product of constant factors ω_c and T is a constant value and in the argument of the sine in (38) it serves as a kind of "pedestal", displacing the level of the argument of the function (38). Since the function (38) is determined by the sequence of phase pulses, following "in tact" with the transmitted information symbols, it actually contains information about the boundaries of CPFM parcels. To select borders of parcels it is sufficient to use intersection moments of the function (38) with the defined (for example, zero) level. This is confirmed by the simulation results. In Fig. 5 below the upper diagram shows the sequence of the first CPFM-differences modulated by random sequence of sign-alternating binary symbols. The lower diagram shows the shape of the processing result according to expression (38). Registering moments of intersecting the zero level (pulses marked with asterisks) allows to select the beginning and the end of a character.

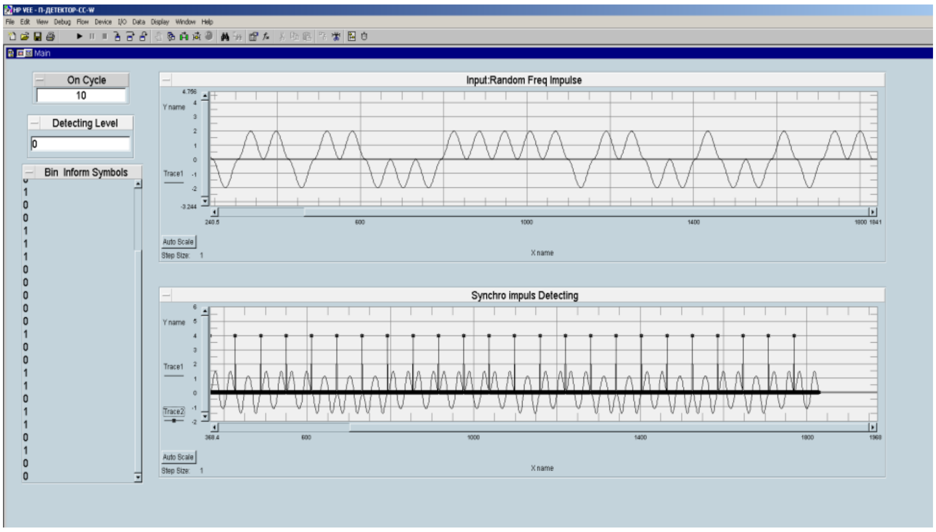

Figure 5 – Illustration of a sync pulse extractor for CMP signal

Thus, in accordance with formulas (36) ... (38), the noncoherent synchronization signal extractor comprises:

1. Two multipliers, the first multiplier is applied to the signal and its copy delayed for the symbol duration, and the second multiplier is applied to the signal and through the $\frac{\pi}{2}$ – phase shifter – to a copy delayed for the duration of the symbol.

2. To suppress the components with doubled frequency, multipliers outputs are fed to the LPF.

3. LPF outputs are multiplied with each other.

4. The synchronization signal is extracted on the basis of registration of points of intersection of the result of multiplication with zero level.

Frequency-selective properties of the active filter. Selective properties of active filters are determined by filtering (averaging) integrators in (*X*, *Y*) channels of correlators. Indeed, when applying the signal $\cos\omega_{\tilde{n}}t$ and the reference oscillation $\cos\omega_0 t$ (with frequency $\omega_0 = \omega_c + \Delta\omega$, shifted relative to the frequency of the signal by the value of $\Delta\omega$) to the inputs of the multiplier, on integrator's output we obtain

$$I_Y = \frac{1}{T}\int_0^T \cos\omega_c t \cos\left(\omega_c + \Delta\omega\right)t\, dt = \frac{1}{2T}\int_0^T \cos(2\omega_c + \Delta\omega)t\, dt + \frac{1}{2T}\int_0^T \cos\Delta\omega t\, dt =$$

$$= \frac{1}{2T\left(2\omega_c + \Delta\omega\right)}\sin\left(2\omega_c + \Delta\omega\right)T + \frac{1}{2\Delta\omega T}\sin\Delta\omega T\,. \quad (39)$$

When processing high-frequency signals $(\omega_c T >> 1)$ the first summand in (39) can be neglected. Then the filtering properties of the AF's correlators are determined by the second summand $\frac{\sin\Delta\omega T}{2\sin\Delta\omega T}$, which turns to zero when frequency of the correlator's reference oscillations is being detuned

$$\Delta\omega = \frac{k\pi}{T} \quad (k = \pm(1,2,3\ \ldots)). \quad (40)$$

Analysis shows that the correlator in the channel *X* has the same properties of selectivity. The selective properties of AF were tested experimentally on the filter model developed using the visual programming software – Agilent HPVEE. The simulation results are shown in Fig. 6. To determine the selective properties of the AF, to the input of the filter (implementing the algorithm (33) with a frequency of the reference oscillator $F_0 = 2000$ Hz) it was given a CPFM signal with a frequency F_c changed in a range (1500–2500) Hz. Fig. 5 shows absolute output values of the AF (33). In this kind of "frequency" characteristic, selective properties of the investigated demodulator are visible, that allowed the author [7] to call it a "filter":

– The presence of the main peak at the frequency of the reference oscillator $F_0 = 2000$ Hz;

– The presence of zeros, which locations correspond to the periodically repeating frequencies (40).

About the noise immunity of demodulation by the active filter. Avoiding the use of coherent demodulation techniques and moving to non-coherent detection can cause a decreasing of the noise immunity and requires a detailed analysis. Such an analysis of noise immunity of noncoherent DPSK demodulation is made in [6] on the basis of the monograph [7]. It was made the comparison of energy costs for providing a given error probability for a coherent and optimal noncoherent demodulation techniques of DFM-2 signals. It is shown that the energy loss of the optimum coherent detection does not exceed 1.0 dB in a wide range of variation of the error probability. This was the reason for the author of the monograph [7, p.88] to declare that this data "... quantitatively prove an important (for the theory of DPSK) thesis that for a high-quality reception of DFM it is not necessarily to have a coherent reference oscillation ...". In addition to the development of noncoherent algorithms performed in this article it is necessary, on the basis of theoretical researches and modeling, to prove or disprove the author's statement [7] about a high noise immunity of noncoherent demodulator (active filter) with reference to CPFM signals.

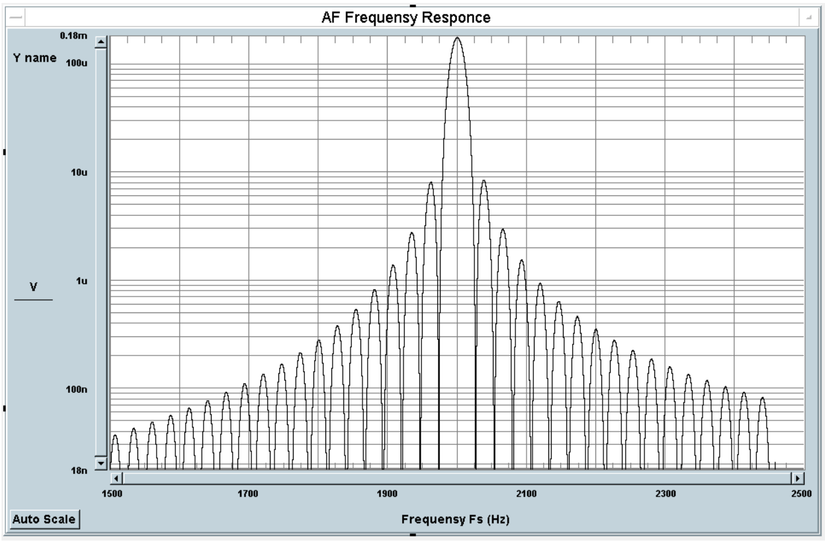

Figure 6 – The dependence of the response at the AF output (with the reference oscillator frequency $F_0 = 2000$ Hz) on the frequency of the signal changed in the range (1500–2500) Hz. Vertical scale is logarithmic

CONCLUSION

1. It is shown that signals of CPFM have the properties of discrete differential modulation in which the transmitted information is put into the changing of the parameters of signals following each other.

2. Noted differential properties allow, in the CPFM demodulation process, to suppress the phase distortions introduced by the quasi-stationary channel with fadings.

3. On the basis of usage of the differential properties of CMFP signals, the following is developed:

3a. A simple algorithm for optimal noncoherent reception of CPFM signals;

3b. An easily implemented method of synchronization signal generation. The method is based on the use of noncoherent signal processing procedures that do not require the use of prior information about the frequency and the phase of the FM signal. This allows to recommend the developed method as a part of noncoherent algorithms of demodulation of CPFM signals.

4. The noncoherent demodulator algorithm, such as claimed in 3a, has the property of frequency selectivity. That allowed to call such a demodulator as the "active filter".

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