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ESTIMATION OF CORRELATION MATRIX OF OBSERVATIONS AT THE FIXED SIGNAL LEVEL BY MAXIMUM LIKELIHOOD CRITERION

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ОЦІНКА КОРЕЛЯЦІЙНОЇ МАТРИЦІ СПОСТЕРЕЖЕНЬ ПРИ ФІКСОВАНОМУ РІВНІ СИГНАЛУ ЗА КРИТЕРІЄМ МАКСИМАЛЬНОЇ ПРАВДОПОДІБНОСТІ

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Abstract. *Based on the fundamental concepts of multivariate complex statistical analysis and matrix theory, estimation of correlation matrix of observations is obtained at the fixed signal level by maximum likelihood criterion in information system with adaptive antenna array. This estimate has been proven to be consistent and sufficient statistics for a correlation matrix of observations in the case of additive Gaussian background noise.*

Key words: *correlation matrix, Gaussian multivariate complex process, probability density, positive definite matrix, Hermitian form, characteristic function, sample estimate, consistent estimate, sufficient statistics.*

Анотація. *Виходячи з основних понять багатомірного комплексного статистичного аналізу та теорії матриць, отримана оцінка кореляційної матриці спостережень при фіксованому рівні сигналу за критерієм максимальної правдоподібності в інформаційній системі з адаптивною антенною решіткою. Доведено, що ця оцінка є спроможною та достатньою статистикою для кореляційної матриці спостережень у випадку прийому сигналу на фоні адитивного гаусівського шуму.*

Ключові слова: *кореляційна матриця, гаусівський багатомірний комплексний процес, щільність ймовірностей, додатно-визначена матриця, ермітова форма, характеристична функція, вибіркова оцінка, спроможна оцінка, достатня статистика.*

INTRODUCTION

Input signal in information system with adaptive antenna array remains one of the main methods of solution of the difficult tasks of signal detection and estimation of its parameters. To solve these problems the correlation matrix $\mathbf{R}$ of vector processes $\mathbf{y}(t)$, which is a mix of the useful signal $\mathbf{s}(t)$ and the additive noise component $\mathbf{n}(t):\mathbf{y}(t)=\mathbf{s}(t)+\mathbf{n}(t)$, is used.

The asymptotic form of correlation matrix of process $\mathbf{y}(t)$ is set as follows

$$\mathbf{R}=\lim_{T\to\infty}\frac{1}{T}\int_0^T \mathbf{y}(t)\cdot\mathbf{y}^{\mathrm{H}}(t)dt,$$

where $(\cdot)^{\mathrm{H}}$ denotes the sign of Hermitian conjugate and *T* denotes the observation time.

Such matrix is Hermitian, positive definite. At Gaussian statistics of jamming's and external noises it contains all information on the processes observed in reception channels of adaptive antenna array [1].

The exact correlation matrix $\mathbf{R}$ can be obtained only from corresponding theoretical models, so in practice instead of it the following estimate is used

$$\hat{\mathbf{R}} = \frac{1}{L}\sum_{j=1}^{L}\mathbf{y}(L)\cdot\mathbf{y}^{\mathrm{H}}(L). \qquad (1)$$

This estimate is obtained from a finite number of training samples of the input vector process in reception channels of adaptive antenna array and, as a result, it depends on the size L of this sample [2].

The fact that estimate (1) is consistent and sufficient statistic for correlation matrix of observations $\mathbf{R}$ was proved in the case when the input vector process is a steady multivariate real Gaussian process [3,4] or a steady multivariate complex Gaussian process with a zero mean [5]. The last is fair in the case when the input vector process contains only a noise component in reception channels. However, in the presence of a useful signal, the corresponding vector process should be considered as a steady multivariate complex Gaussian process with mean value $\mathbf{s}(t) \neq 0$. According to the modern research conducted worldwide, estimate (1) is also used in the case when the useful signal at entrance vector process is present. But, the legitimacy of such use, in sense of consistency and sufficiency of this statistics, demands an additional proof.

The aim of work is to prove the fact that the estimate (1) is a consistent and sufficient statistic for correlation matrix of observations $\mathbf{R}$ in case of a steady multivariate complex Gaussian process with mean value $\mathbf{s}(t) \neq 0$.

ESTIMATION OF CORRELATION MATRIX AT THE FIXED SIGNAL LEVEL

The set of complex amplitudes of the input process in separate reception channels of adaptive antenna array with N elements at the any instant of t can be presented in the form of a N-dimensional column vector

$$\mathbf{y}(t) = \left[y_1(t)y_2(t)\mathrm{K}\, y_N(t)\right]^{\mathrm{T}}.$$

In practice, the input process is observed in discrete instants. By $\mathbf{y}(j);\ j=\overline{1,L}$ we define the L available samples of vector $\mathbf{y}(t)$, corresponding to the L instants which are selected after identical intervals of Δt. The time interval between the samples is set in the way that samples are statistically independent.

Let's assume that the width of frequency spectrum of the accepted fluctuations significantly exceeds a band Δf of the receiver and within this band its spectrum can be considered as uniform. If the interval $\Delta t \geq 1/\Delta f$, N-dimensional random samples of the input process $\mathbf{y}(1),\mathbf{y}(2),...,\mathbf{y}(L)$ are statistically independent among themselves [4].

Let's consider that L independent identically distributed N-dimensional complex Gaussian random values $\mathbf{y}(j);\ j=\overline{1,L}$ are samples of the size L from set

$$p(\mathbf{y}/\mathbf{s};\mathbf{R}) = \frac{1}{\pi^N \det\mathbf{R}} e^{-(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})}. \qquad (2)$$

Then, their collateral probability density which is called likelihood function [6], has the following form

$$\Lambda(\mathbf{R}) = p(\mathbf{y}(1),\mathbf{y}(2),...,\mathbf{y}(L)) = \prod_{j=1}^{L} p(y(j)) = \prod_{j=1}^{L}\frac{1}{\pi^N \det\mathbf{R}} e^{-(\mathbf{y}(j)-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}(j)-\mathbf{s})} =$$

$$= \frac{1}{\left(\pi^N \det\mathbf{R}\right)^L}\cdot\exp\left(-\sum_{j=1}^{L}(\mathbf{y}(j)-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}(j)-\mathbf{s})\right). \qquad (3)$$

As $\mathbf{y}(j);\ j=\overline{1,L}$ are fixed values of sample and correlation matrix $\mathbf{R}$ depends on existence of the useful signal $\mathbf{s}(t)$, the likelihood function can be considered as function from $\mathbf{R}$. Those values $\mathbf{R}$ at which function $\Lambda(\mathbf{R})$ reaches a local maximum is the most likely estimate $\hat{\mathbf{R}}$ for correlation matrix $\mathbf{R}$. In addition, several auxiliary statements need to be proved.

Lemma 1. If $\mathbf{A}$ denotes an Hermitian positive define matrix, then the characteristic function $\mathbf{\Phi}_{\mathbf{R},\mathbf{A}}(\theta)$ with respect to the density (2) of the Hermitian form

$$E_{\mathbf{A}} = (\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})$$

is

$$\mathbf{\Phi}_{\mathbf{R},\mathbf{A}}(\theta) = \det{}^{-1}(\mathbf{I}-i\theta\mathbf{RA}). \tag{4}$$

Proof: Since

$$\int_{\mathbf{y}} p(\mathbf{y}/\mathbf{s};\mathbf{R})\,d\mathbf{y} = 1,$$

from (2) we receive

$$\int_{\mathbf{y}} \frac{1}{\pi^N \det\mathbf{R}} e^{-(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})} d\mathbf{y} = 1.$$

Consequently,

$$\det\mathbf{R} = \int_{\mathbf{y}} \frac{1}{\pi^N} e^{-(\mathbf{y}-\mathbf{s})^{H}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})} d\mathbf{y}. \tag{5}$$

Characteristic function $\Phi_{\mathbf{R},\mathbf{A}}(\theta)$ of Hermitian form $E_{\mathbf{A}}$ is an expectation of a random value of $e^{i\theta E_{\mathbf{A}}}$ [6].

Thus, taking in consideration (5) and the properties of determinants [7], we receive

$$\Phi_{\mathbf{R},\mathbf{A}}(\theta) = M\left[e^{i\theta(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})}\right] =$$

$$= \int_{\mathbf{y}} e^{i\theta(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})} \frac{1}{\pi^N \det\mathbf{R}} e^{-(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})} d\mathbf{y} = \det{}^{-1}\mathbf{R}\cdot\det\left(\mathbf{R}^{-1}-i\theta\mathbf{A}\right)^{-1} =$$

$$= \det{}^{-1}\mathbf{R}\cdot\det{}^{-1}\left(\mathbf{R}^{-1}-i\theta\mathbf{A}\right) = \det{}^{-1}\left(\mathbf{R}\cdot\mathbf{R}^{-1}-i\,\theta\mathbf{RA}\right) =$$

$$= \det{}^{-1}\left(\mathbf{I}-i\,\theta\mathbf{RA}\right). \tag{6}$$

Lemma 2. If $\mathbf{A}$ denotes an Hermitian positive define matrix, then

$$M\left[(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\right] = tr(\mathbf{PA}). \tag{7}$$

Proof: According to the definition

$$M\left[(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\right] = \int_{\mathbf{y}} (\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\,p(\mathbf{y}/\mathbf{s};\mathbf{R})\,d\mathbf{y} =$$

$$= \int_{\mathbf{y}} (\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\cdot\frac{1}{\pi^N \det\mathbf{R}} e^{-(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})} d\mathbf{y}.$$

From (6) we receive

$$\frac{d}{d\theta}\Phi_{\mathbf{R},\mathbf{A}}(\theta)\bigg|_{\theta=0} = \int_{\mathbf{y}} \frac{d}{d\theta}\left[e^{i\theta(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})}\right]\bigg|_{\theta=0}\cdot\frac{1}{\pi^N \det\mathbf{R}} e^{-(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})} d\mathbf{y} =$$

$$=\int_{\mathbf{y}}[i(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\,e^{i\theta(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})}]\Big|_{\theta=0}\cdot\frac{1}{\pi^{N}\det\mathbf{R}}e^{-(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})}d\mathbf{y}=$$

$$=i\int_{\mathbf{y}}(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\frac{1}{\pi^{N}\det\mathbf{R}}e^{-(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{R}^{-1}(\mathbf{y}-\mathbf{s})}d\mathbf{y}=i\,M\left[(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\right].$$

Thus, taking in consideration (4), the expression (7) can be presented in the form

$$M\left[(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\right]=-i\frac{d}{d\theta}\Phi_{\mathbf{R},\mathbf{A}}(\theta)\Big|_{\theta=0}=-i\frac{d}{d\theta}\left[\det{}^{-1}(\mathbf{I}-i\theta\mathbf{R}\mathbf{A})\right]\Big|_{\theta=0}=$$

$$=-i\left[-\det{}^{-2}(\mathbf{I}-i\theta\mathbf{R}\mathbf{A})\cdot\frac{d}{d\theta}\det(\mathbf{I}-i\theta\mathbf{R}\mathbf{A})\right]\Big|_{\theta=0}=$$

$$=i\,\frac{d}{d\theta}\det(\mathbf{I}-i\theta\mathbf{R}\mathbf{A})\Big|_{\theta=0}. \tag{8}$$

Let matrix elements $\mathbf{RA}=\{\sigma_{jk}\};\ j,k=\overline{1,N}$. So,

$$\det(\mathbf{I}-i\theta\mathbf{R}\mathbf{A})=\begin{vmatrix}1-i\theta\sigma_{11} & -i\theta\sigma_{12} & \cdots & -i\theta\sigma_{1N}\\ -i\theta\sigma_{21} & 1-i\theta\sigma_{22} & \cdots & -i\theta\sigma_{2N}\\ \cdots & \cdots & \cdots & \cdots\\ -i\theta\sigma_{N1} & -i\theta\sigma_{N2} & \cdots & 1-i\theta\sigma_{NN}\end{vmatrix}$$

and, according to the rule of derivation of n-order determinants [8]

$$\frac{d}{d\theta}\det(\mathbf{I}-i\theta\mathbf{R}\mathbf{A})\Big|_{\theta=0}=\begin{vmatrix}-i\,\sigma_{11} & -i\,\sigma_{12} & \cdots & -i\,\sigma_{1N}\\ 0 & 1 & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & \cdots & 1\end{vmatrix}+\cdots+\begin{vmatrix}1 & 0 & \cdots & 0\\ 0 & 1 & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots\\ -i\,\sigma_{N1} & -i\,\sigma_{N2} & \cdots & -i\,\sigma_{NN}\end{vmatrix}=$$

$$=-i\,\sigma_{11}-i\,\sigma_{12}-\ldots-i\,\sigma_{NN}=-i\,tr(\mathbf{R}\mathbf{A}). \tag{9}$$

Then, considering (9), expression (8) takes the form

$$M\left[(\mathbf{y}-\mathbf{s})^{\mathrm{H}}\mathbf{A}(\mathbf{y}-\mathbf{s})\right]=i\left(-i\,tr(\mathbf{R}\mathbf{A})\right)=tr(\mathbf{R}\mathbf{A}).$$

Let's enter a vector of sample averages of the input vector process

$$\mathbf{y}^{*}=\frac{1}{L}\sum_{j=1}^{L}\mathbf{y}(j),$$

and sample correlation matrix

$$\mathbf{R}^{*}=\frac{1}{L}\sum_{j=1}^{L}\mathbf{y}(j)\cdot\mathbf{y}^{\mathrm{H}}(j). \tag{10}$$

Lemma 3. The sample correlation matrix $\mathbf{R}^{*}$ is the consistent estimate of correlation matrix of observations $\mathbf{R}$.

Proof: The sample correlation matrix $\mathbf{R}^{*}$ is the consistent estimate of correlation matrix $\mathbf{R}$ if $\forall\varepsilon>0\quad p\left(\left|\mathbf{R}^{*}-\mathbf{R}\right|>\varepsilon\right)\to 0$ at $L\to\infty$.

To prove that we use Kullback-Leybler divergence [6] (or the relative entropy) between two probability distributions

$$D\left(\mathbf{R}/\mathbf{R}^*\right) = \int_{\mathbf{y}} p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right) \ln \frac{p\left(\mathbf{y}/\mathbf{s};\mathbf{R}^*\right)}{p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right)} d\mathbf{y},$$

as an index of average information from observation in advantage $\mathbf{R}^*$.

As $\mathbf{R}$ and $\mathbf{R}^*$ are positive definite matrixes, $\mathbf{R}^{-1}$and $\left(\mathbf{R}^*\right)^{-1}$ exist and are also positive definite Hermitian matrixes. Then, on the basis of a lemma 2 and formulas (2), we receive

$$D\left(\mathbf{R}/\mathbf{R}^*\right) = \int_{\mathbf{y}} p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right)\left[\ln p\left(\mathbf{y}/\mathbf{s};\mathbf{R}^*\right) - \ln p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right)\right] d\mathbf{y} =$$

$$= \int_{\mathbf{y}} p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right)\left[\ln \frac{\det \mathbf{R}}{\det \mathbf{R}^*} - \left(\mathbf{y}-\mathbf{s}\right)^{\mathrm{H}}\left(\mathbf{R}^*\right)^{-1}\left(\mathbf{y}-\mathbf{s}\right) + \left(\mathbf{y}-\mathbf{s}\right)^{\mathrm{H}}\mathbf{R}^{-1}\left(\mathbf{y}-\mathbf{s}\right)\right] d\mathbf{y} =$$

$$= \ln \frac{\det \mathbf{R}}{\det \mathbf{R}^*} \int_{\mathbf{y}} p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right)\, d\mathbf{y} - \int_{\mathbf{y}} \left(\mathbf{y}-\mathbf{s}\right)^{\mathrm{H}}\left(\mathbf{R}^*\right)^{-1}\left(\mathbf{y}-\mathbf{s}\right) p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right) d\mathbf{y} +$$

$$= \int_{\mathbf{y}} \left(\mathbf{y}-\mathbf{s}\right)^{\mathrm{H}}\mathbf{R}^{-1}\left(\mathbf{y}-\mathbf{s}\right) p\left(\mathbf{y}/\mathbf{s};\mathbf{R}\right) d\mathbf{y} = \ln \frac{\det \mathbf{R}}{\det \mathbf{R}^*} - tr\left(\mathbf{R}\left(\mathbf{R}^*\right)^{-1}\right) + tr\left(\mathbf{R}\mathbf{R}^{-1}\right) =$$

$$= \ln \frac{\det \mathbf{R}}{\det \mathbf{R}^*} - tr\left(\mathbf{R}\left(\mathbf{R}^*\right)^{-1}\right) + tr\left(\mathbf{I}\right).$$

It's apparent that $D\left(\mathbf{R}/\mathbf{R}^*\right) \to 0$, if $\mathbf{R}^* \to \mathbf{R}$ at $L \to \infty$.■

Further, for the sum in the exponent of likelihood function (3), by means of properties of a trace of the sum and a trace of a matrix product [7], we receive

$$\sum_{j=1}^{L}\left(\mathbf{y}(j)-\mathbf{s}\right)^{\mathrm{H}}\mathbf{R}^{-1}\left(\mathbf{y}(j)-\mathbf{s}\right) = \sum_{j=1}^{L} tr\left[\left(\mathbf{y}(j)-\mathbf{s}\right)^{\mathrm{H}}\mathbf{R}^{-1}\left(\mathbf{y}(j)-\mathbf{s}\right)\right] =$$

$$= \sum_{j=1}^{L} tr\left[\mathbf{R}^{-1}\left(\mathbf{y}(j)-\mathbf{s}\right)\left(\mathbf{y}(j)-\mathbf{s}\right)^{\mathrm{H}}\right] = tr\left[\mathbf{R}^{-1}\sum_{j=1}^{L}\left(\mathbf{y}(j)-\mathbf{s}\right)\left(\mathbf{y}(j)-\mathbf{s}\right)^{\mathrm{H}}\right]. \quad (11)$$

Let's notice that

$$\frac{1}{L}\sum_{j=1}^{L}\left(\mathbf{y}(j)-\mathbf{s}\right)\left(\mathbf{y}(j)-\mathbf{s}\right)^{\mathrm{H}} = \frac{1}{L}\sum_{j=1}^{L}\mathbf{y}(j)\cdot\mathbf{y}^{\mathrm{H}}(j) - \frac{2}{L}\sum_{j=1}^{L}\mathbf{y}(j)\cdot\mathbf{s}^{\mathrm{H}} + \mathbf{s}\cdot\mathbf{s}^{\mathrm{H}} =$$

$$= \mathbf{R}^* - 2\mathbf{y}^*\cdot\mathbf{s}^{\mathrm{H}} + \mathbf{s}\cdot\mathbf{s}^{\mathrm{H}} = \mathbf{R}^* + \mathbf{y}^*\cdot(\mathbf{y}^*)^{\mathrm{H}} - 2\mathbf{y}^*\cdot\mathbf{s}^{\mathrm{H}} + \mathbf{s}\cdot\mathbf{s}^{\mathrm{H}} - \mathbf{y}^*\cdot(\mathbf{y}^*)^{\mathrm{H}} =$$

$$= \mathbf{R}^* + \left(\mathbf{y}^*-\mathbf{s}\right)\left(\mathbf{y}^*-\mathbf{s}\right)^{\mathrm{H}} - \mathbf{y}^*\cdot\left(\mathbf{y}^*\right)^{\mathrm{H}}.$$

Then, (11) has the form

$$\sum_{j=1}^{L}\left(\mathbf{y}(j)-\mathbf{s}\right)^{\mathrm{H}}\mathbf{R}^{-1}\left(\mathbf{y}(j)-\mathbf{s}\right) = L\,tr\left[\mathbf{R}^{-1}\mathbf{R}^*\right] +$$

$$+ L\left(\mathbf{y}^*-\mathbf{s}\right)^{\mathrm{H}}\mathbf{R}^{-1}\left(\mathbf{y}^*-\mathbf{s}\right) - L\,\mathbf{y}^*\mathbf{R}^{-1}\left(\mathbf{y}^*\right)^{\mathrm{H}}. \quad (12)$$

Theorem. The sample correlation matrix $\mathbf{R}^*$ is a sufficient statistics for correlation matrix of observations $\mathbf{R}$.

Proof: According to the sign of factorization [3], statistics is sufficient if likelihood function can be presented as a product of two non-negative multipliers, when one of which depends on this statistics and estimated parameters, but another one doesn't depend on this statistics.

When substituted (12) in likelihood function (3), we receive

$$\Lambda(\mathbf{R}) = \frac{1}{\left(\pi^N \det \mathbf{R}\right)^L} e^{-Ltr\left[\mathbf{R}^{-1}\mathbf{R}^*\right] - L\left(\mathbf{y}^* - \mathbf{s}\right)^{\mathrm{H}} \mathbf{R}^{-1}(\mathbf{y}^* - \mathbf{s}) + L\mathbf{y}^*\mathbf{R}^{-1}(\mathbf{y}^*)^{\mathrm{H}}} =$$

$$= \frac{1}{\left(\pi^N \det \mathbf{R}\right)^L} e^{-L\left(\mathbf{y}^* - \mathbf{s}\right)^{\mathrm{H}} \mathbf{R}^{-1}\left(\mathbf{y}^* - \mathbf{s}\right) + L\mathbf{y}^*\mathbf{R}^{-1}\left(\mathbf{y}^*\right)^{\mathrm{H}}} \cdot e^{-Ltr\left[\mathbf{R}^{-1}\mathbf{R}^*\right]}. \quad (13)$$

Thus, sample correlation matrix $\mathbf{R}^*$ is a consistent and sufficient estimate of correlation matrix of observations $\mathbf{R}$.

CONCLUSIONS

The sample correlation matrix $\mathbf{R}^*$ (10) coincides with the estimate $\hat{\mathbf{R}}$, therefore the legitimacy of estimate (1) at the fixed level of the useful signal $\mathbf{s}(t) \neq 0$ is proved.

Besides, if the useful signal is absent $\mathbf{s}(t) = 0$, likelihood function (13) has the form

$$\Lambda(\mathbf{R}) = \frac{1}{\left(\pi^N \det \mathbf{R}\right)^L} e^{-Ltr\left[\mathbf{R}^{-1}\mathbf{R}^*\right]},$$

that completely coincides with the result received in [5].

In the expression of likelihood function (13)

$$\mathbf{y}^* \mathbf{R}^{-1}\left(\mathbf{y}^*\right)^{\mathrm{H}} \geq \left(\mathbf{y}^* - \mathbf{s}\right)^{\mathrm{H}} \mathbf{R}^{-1}\left(\mathbf{y}^* - \mathbf{s}\right) \succ 0,$$

as $\mathbf{R}$ is a positive definite and $\left\|\mathbf{y}^*\right\| \geq \left\|\mathbf{y}^* - \mathbf{s}\right\| \succ 0$. It means that likelihood function $\Lambda(\mathbf{R})$ reaches the maximum value when $\mathbf{R}^{-1}\mathbf{R}^* \to \mathbf{I}$, that is when $\mathbf{R}^* = \hat{\mathbf{R}} \to \mathbf{R}$ at $L \to \infty$. So, the estimate (1) is the most plausible estimate also in case when the useful signal at vector process $\mathbf{y}(t)$ is present.

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